Exercise 1 (4 points)

a) Let $L/K$ be an algebraic and separable field extension. Show that $\Omega_{L/K} = 0$.

b) Let $K$ be a field of characteristic $p > 0$ and $c$ be an element of $K$ which is no $p$-power in $K$. Let $L = K[X]/(X^p - c)$ be the splitting field of $f := X^p - c$. Show that $\Omega_{L/K} \cong L$.

Exercise 2 (4 points)
Let $K$ be a field of characteristic 0 and $A := K[X_1, \ldots, X_n]$. Show that $H^1_{\text{dR}}(A) = 0$.

Exercise 3 (4 points)
Determine $\Omega_{A/R}$ for $A := R[X,Y]/(XY)$.

Exercise 4 (4 points)
Let $A$ be an $R$-algebra and define an $A$-homomorphism $\Phi : A \otimes_R A \to A$ by $\Phi(a \otimes b) := ab$. Let $I$ be the kernel of $\Phi$. Show that the map
\[ \varphi : \Omega_{A/R} \to I/I^2, \quad da \mapsto (1 \otimes a - a \otimes 1) + I^2 \]
is an isomorphism of $A$-modules.

Hint: Consider the ring $B := \{(a, x) : a \in A, x \in \Omega_{A/R}\}$ in which the operations are given by $(a_1, x_1) + (a_2, x_2) := (a_1 + a_2, x_1 + x_2)$ and $(a_1, x_1) \cdot (a_2, x_2) := (a_1a_2, a_1x_2 + a_2x_1)$ ($a_1, a_2 \in A, x_1, x_2 \in \Omega_{A/R}$). Use the $R$-algebra-homomorphism $\psi : A \otimes_R A \to B, a \otimes b \mapsto (ab, ab)$.

Solutions to be handed in on Tuesday, 20.5.2008, at the beginning of the problem session in S12.