Exercise 56: Consider \( f(x) = \sqrt[3]{2x+2} \) for \( x \geq -1 \).

(a) Determine the Taylor polynomial \( p(x) \) of degree 2 of \( f \) expanded about \( x_0 = 3 \).

(b) Find an upper bound on the absolute value of the Lagrange form of the remainder \( f(x) - p(x) \) that is valid for any \( x \) in the interval \([\frac{-1}{2}, 3]\).

Exercise 57: Determine all derivatives \( f^{(n)} \), \( n \in \mathbb{N}_0 \), of the function \( f \), and specify the Taylor series of \( f \) expanded about \( x_0 = 0 \):

(a) \( f(x) = \sin 3x \), \( x \in \mathbb{R} \)

(b) \( f(x) = \sqrt{1 + x} \), \( |x| \leq 1 \).

Where do these series converge?

Exercise 58: Compute the following integrals using integration by parts:

(a) \( \int_0^1 t \cdot \arctan(t) \, dt \), \hspace{1cm} (b) \( \int_0^\frac{\pi}{2} \cos^3(t) \cdot \cos(t) \, dt \).

Use also integration by parts to compute the following indefinite integrals:

(c) \( \int \sin^2(x) \, dx \), \hspace{1cm} (d) \( \int x^2 \cdot \ln(x) \, dx \).

Exercise 59: Determine the following integrals using integration by substitution:

(a) \( \int \frac{dx}{x \ln x} \) on \((1, \infty)\), \hspace{1cm} (b) \( \int \frac{x}{\sqrt{x^2 - 1}} \, dx \) on \((1, \infty)\).

Determine the following integral using a suitable substitution and afterwards integration by parts:

(c) \( \int_1^4 \arctan(\sqrt{x} - 1) \, dx \).

Exercise 60: Let \( c \) be some time point, \( c \geq 4 \). A pool of water is supplied through four feed pipes. At each moment there is a different water amount passing through the pipes. At time \( t = 0 \) the pool contains 100 liters of water. During the first second \( \frac{\ln 2}{2} \) liters of water per second flow into the pool through the first pipe and \( 9te^{-3t} + \frac{2}{1+t^2} \) liters per second through the second pipe at a time \( t \). For example at the time \( t = 0.001 \) pour in \( \frac{\ln 2}{2000} \) liters per second through pipe number one. During the first four seconds \( \frac{1}{2\sqrt{t}} \) liters of water per second leave the pool through pipe number three. Starting at a time \( c \) the water enters the pool through the fourth pipe at a rate of \( \frac{1}{2} \) liters per second.

How much water will the pool contain at the time \( t = 2c \)?

Due date: Please hand in your homework on Monday, January 28, 12:00, into the AM1-box in the student office in the International Department.
Exercise T35: Compute the following antiderivatives using integration by parts:

(a) $\int x^2 \sin x \, dx$,  
(b) $\int \arctan \frac{1}{x-1} \, dx$,  
(c) $\int (\ln y)^2 \, dy$.

(d) Moreover, show that the following equation holds: $\int_0^{2\pi} \cos^2 x \, dx = \int_0^{2\pi} \sin^2 x \, dx = \pi$.

Exercise T36: Calculate the following integrals using suitable substitutions:

(a) $\int_0^{2\pi} \cos(x) \cdot e^{\sin(x)} \, dx$,  
(b) $\int \frac{2x + 7}{x^2 + 7x + 3} \, dx$,  
(c) $\int \frac{\cos(\ln(x))}{x} \, dx$,  
(d) $\int \frac{1 + \ln(x)}{x - x \ln(x)} \, dx$.

Exercise T37:
A 30 cm long bar of brass has as cross section a circle with diameter 4 mm, which is flattened at one side by 1 mm (see sketch). Brass has the density 8.4 g/cm$^3$. How heavy is the bar?

For detailed information regarding this course please check the page
http://www.math.kit.edu/iag6/lehre/am12012w/en

Tutorial date: Wednesday, January 23, 2013, 14:00-15:30 pm.