Worksheet No.3
Advanced Mathematics I

Exercise 11: Consider the complex numbers \( z_1 = 1 + i \), \( z_2 = 2 - 3i \), \( z_3 = \sqrt{3} + i \). Compute

(a) the real and imaginary part of \( \bar{z}_j, -z_j, z_j\bar{z}_j, \frac{1}{z_j}, z_j - \bar{z}_j \) and \( |z_j| \), \( j = 1, 2 \), as well as the real and imaginary part of \( \frac{z_1}{z_1 + z_2}, \) \( z_1^3z_2^2 \).

(b) the representation of \( z_3 \) by polar coordinates \((r, \varphi)\), where \( \varphi \) represents the principal value of the argument of \( z_3 \).

Exercise 12: Sketch the set of all complex numbers \( z \) that satisfy the respective conditions:

(a) \( |\text{Re}z| + |\text{Im}z| \leq 4 \),
(b) \( |z|^2 \leq 2\text{Re}z \),
(c) \( z^4 + (2i + 2)z^2 + 4i = 0 \).

Exercise 13: Compute the real and imaginary part of the complex number \( z = (1 + i)^5 \)

(a) using the binomial theorem,
(b) using the representation of \( 1 + i \) by polar coordinates.
(c) Which method above is easier to compute \( w = (1 - i)^{17} \)?
Determine \( \text{Re}w \) and \( \text{Im}w \).

Exercise 14: Determine all values of \( \alpha \in \mathbb{C} \) for which the following system of linear equations

\[
\begin{align*}
(ia - i)z_1 + (ia^2 - i)z_2 - (1 + i)z_3 &= -2 - 2i \\
(i - 1)z_1 + (i - 1)z_2 - iz_3 &= -1 - i \\
z_1 + z_2 - z_3 &= -1
\end{align*}
\]

(a) has no solutions,
(b) has infinitely many solutions. Determine these solutions.

Exercise 15: Prove by mathematical induction on \( n \in \mathbb{Z}, n \geq 0 \), the following statement:

\[
\sum_{k=0}^{2n} i^k k = \begin{cases} 
 n(1 - i), & n \text{ even} \\
 -(n + 1) + ni, & n \text{ odd}
\end{cases}
\]

Due date: Please hand in your homework on Monday, November 12, 12:00, into the AM1-box in the student office in the International Departement.
Exercise T7:

(a) Consider the complex numbers

\begin{align*}
&i) \quad z = 3 - i \\
&ii) \quad z = 3 + 4i.
\end{align*}

Plot each of the numbers $z, -z, z\overline{z}, \frac{1}{z}, z - \overline{z}, |z|$ in the complex plane by splitting them into their real and imaginary part.

(b) Compute the representation in polar coordinates $(r, \text{Arg} \ z)$ of the complex numbers

\begin{align*}
&i) \quad -\frac{1}{4} + \frac{1}{4}\sqrt{3}i, \\
&ii) \quad -1 - i.
\end{align*}

Exercise T8: Sketch the set of all complex numbers $z$ that satisfy the respective conditions:

(a) $|3z - 1 + 2i| \leq 2$, \\
(b) $|z - z_0| = |z - z_1|$ for $z_0 = 1 - i, z_1 = 2 + i$.

Exercise T9: Determine the real and imaginary part of all solutions $w \in \mathbb{C}$ of the equations

\begin{align*}
&\text{(a) } w^2 = -5 + 12i, \\
&\text{(b) } w^2 + 6iw - 6 = 4i.
\end{align*}

For detailed information regarding this course please check the page

http://www.math.kit.edu/iag6/lehre/am12012w/en

Tutorial date: Wednesday, November 7, 2012, 14:00-15:30 pm.