Exercise Sheet No. 12
Advanced Mathematics I

Exercise 56: Find all global extrema of the following function on the interval \([0, 2]\)

\[ f(x) = \ln(x+1) - \frac{x^2}{2+2x}. \]

Exercise 57:
(a) Calculate the following anti-derivatives on \((1, \infty)\) using integration by substitution:

\[ (1) \int \frac{dx}{x \ln x}, \quad (2) \int \frac{x}{\sqrt{x^2-1}} \, dx. \]

(b) Calculate the following integral using integration by substitution first and integration by parts afterwards

\[ \int_{1}^{4} \arctan \left( \sqrt{x-1} \right) \, dx. \]

Hint: The derivative of \(\arctan\) was calculated in Exercise 49.

(c) Calculate the following integrals using integration by parts

\[ (1) \int_{0}^{\pi/2} \cos(t)^3 \cdot \cos(t) \, dt, \quad (2) \int (\sin(x))^2 \, dx, \quad (3) \int x^2 \cdot \ln(x) \, dx. \]

Exercise 58: Find the derivative \(F'\) of the following function \(F\)

\[ F : [0, 1] \to \mathbb{R}, \quad F(x) := \int_{\ln(x)}^{x^2} \sin(\cos(t)) \, dt. \]

Exercise 59: Consider the value \(J\) of the integral \(J = \int_{\pi/6}^{\pi/2} \frac{\sin(x)}{x} \, dx\).

Prove the following lower and upper bounds on \(J\)

(a) \(\ln \sqrt{3} < J < \ln 3\), \hspace{1cm} (b) \(\frac{\sqrt{3}}{\pi} < J < \frac{3\sqrt{3}}{\pi}\).

Exercise 60: For each of the following functions, find all anti-derivatives

(a) \(f : (1, \infty) \to \mathbb{R}, \quad f(x) = \frac{1}{x \ln(x) \ln(\ln(x))}\), \hspace{1cm} (b) \(g : (0, \infty) \to \mathbb{R}, \quad g(x) = \left( \frac{\ln(x)}{x} \right)^2\),

(c) \(h : \mathbb{R} \to \mathbb{R}, \quad h(x) = \frac{x^{\arctan(x)}}{(1 + x^2)^{3/2}}\).

No submission: Please do not submit solutions as they will not be corrected.

Note that the material may be part of the exam though.
Tutorial No. 12
Advanced Mathematics I

Exercise T34: Prove the following inequalities for all $x \in [-1, 1]$:

(a) $6x^3 + 3x^2 > 4x - 1$,

(b) $\arcsin(x) \leq \frac{x}{2} + 2x\sqrt{1 - x^2}$.

Exercise T35: Calculate the following anti-derivatives using integration by parts

(a) $\int x^2 \sin(x)dx$,

(b) $\int \arctan\left(\frac{1}{x-1}\right) dx$,

(c) $\int (\ln y)^2 dy$.

(d) Show that the following equation holds

$$\int_0^{2\pi} (\cos(x))^2 dx = \int_0^{2\pi} (\sin(x))^2 dx = \pi.$$

Exercise T36: Calculate the following integrals using integration by substitution

(a) $\int_0^{\frac{\pi}{2}} \cos(x) \cdot e^{\sin(x)} dx$,

(b) $\int \frac{2x + 7}{x^2 + 7x + 3} dx$,

(c) $\int \frac{\cos(\ln(x))}{x} dx$,

(d) $\int \frac{1 + \ln(x)}{x - x \ln(x)} dx$.

Please note the following information regarding the final AM 1 exam:

- The exam takes place on Saturday, February 21st, 9:00 - 11:00.
- You need to register online (campus.studium.kit.edu) between 03.02. and 16.02.2015.
- A seating plan will be published on our website few days before the exam.

For detailed information regarding this course and the exam visit the following web page:

www.math.kit.edu/iag6/lehre/am12014w/en

Tutorial: Wednesday, 4th February 2015, 15:45 PM