Exercise 11:

(a) Let $z, w \in \mathbb{C}$. Calculate $\text{Re}(zw)$ in two ways, using

(i) Cartesian coordinates, i.e. $z = x + iy$, $w = a + ib$ for $x, y, a, b \in \mathbb{R}$,

(ii) polar coordinates $z = r(\cos(\varphi) + i\sin(\varphi))$, $w = q(\cos(\psi) + i\sin(\psi))$ with $r, q \in [0, \infty)$, $\varphi, \psi \in (-\pi, \pi]$.

(b) Determine the real and imaginary parts, the absolute values and the principal arguments of all the $z \in \mathbb{C}$ satisfying

$$z^2 = \frac{3}{1-3i} - \frac{1}{3+i}.$$ 

Exercise 12: Give a sketch of each of the following subsets of $\mathbb{C}$:

$K = \{ z \in \mathbb{C} : |2z - 4i|^2 = 16 \}$,

$R = \{ z \in \mathbb{C} : 1 \leq |z - 3 + 4i| \leq 3 \}$,

$G = \{ z \in \mathbb{C} : |z - 1 + i| = |z - 2 - i| \}$,

$H = \{ z \in \mathbb{C} : \text{Re}(z \cdot (1 - i)) \geq 0 \}$.

Exercise 13: Find all solutions $z \in \mathbb{C}$ of the following equations by completing the square

(a) $z^2 + (-10 + 4i)z + 70 - 20i = 0$,  

(b) $z^2 - 6z - 3 + i(4z + 6) = 0$.

Exercise 14: Let $a_n := \frac{n}{\sqrt{n^2 + 1}}$ for all $n \in \mathbb{N}$. Consider the sequence $(a_n)_{n=1}^{\infty}$.

(a) Show that the sequence $(a_n)$ converges to 1.

(b) Find some $N \in \mathbb{N}$ such that $|a_n - 1| < \frac{1}{200}$ for all $n \geq N$.

Exercise 15: Consider the following sequences $(a_n)$, $(b_n)$, $(c_n)$, $(d_n)$ for $n \in \mathbb{N}$ and determine whether they are (1) convergent or not, (2) bounded or unbounded and (3) tending to $\pm\infty$ or not

(a) $a_n = \frac{1 + n + n^2}{n(n + 1)}$ ,  

(b) $b_n = \frac{1 + n + n^2}{n + 1}$ ,  

(c) $c_n = \frac{1}{1 + (-2)^n}$ ,  

(d) $d_n = \frac{1 + (-2)^n}{1 + 2^n}$ .

Due date: Your written solutions are due by 12:30 P.M. on Monday, 24th November 2014. Please put them into the box in the student office.
Exercise T7: Find all solutions \( z \in \mathbb{C} \) of the following equations by means of completing the square

\[
\begin{align*}
(a) & \quad 3z^2 + 24z - 78 \\
(b) & \quad z^2 + (2 - 2i\sqrt{2})z - 7 - i(8 + 2\sqrt{2}) = 0
\end{align*}
\]

Exercise T8:

(a) Let \( z_1 = 1 + i, z_2 = 2 - 3i \).

Calculate the real and imaginary parts as well as the absolute value of the following complex numbers

\[
\frac{z_2z_1}{z_1 \bar{z}_1}, \quad \frac{z_2}{z_1}, \quad \frac{1}{z_1 + z_2}, \quad \frac{\bar{z}_1}{z_1}.
\]

(b) Give a sketch of the following subset of \( \mathbb{C} \)

\[
K = \{ z \in \mathbb{C} : |z - i(1 + i)| = 1 \}.
\]

(c) Give the principal argument of \((\frac{1}{2} + i\frac{1}{2}\sqrt{3})^n\) for all \( n \in \mathbb{N} \).

Exercise T9: Consider the sequence \((a_n)_n\) with \( a_n = \frac{n-1}{n+1} \) for all \( n \in \mathbb{N} \).

(a) Find an index \( N \in \mathbb{N} \) such that \( |a_n - 1| < \varepsilon \) for all \( n \geq N \) if

\[
\begin{align*}
(i) & \quad \varepsilon = \frac{1}{4}, \\
(ii) & \quad \varepsilon = \frac{1}{100}, \\
(iii) & \quad \varepsilon > 0 \text{ is arbitrary.}
\end{align*}
\]

(b) Does the sequence \((a_n)_n\) converge? Find the limit in case it converges.

For detailed information regarding this course visit the following web page:

www.math.kit.edu/iag6/lehre/am12014w/en