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Exercise Sheet No. 1 Advanced Mathematics I

Exercise 1: Consider the following sets of real numbers: $A = [-4, -1]$, $B = \{-3, -2, 0, 3\}$ and $C = [-2, 4]$.

- (a) Determine the sets $A \cup B$, $A \cap B$ and $(A \setminus C) \cup (C \setminus A)$.
- (b) Find a maximal set M satisfying $(M \setminus A) \subseteq (B \cup C)$.

Exercise 2: Solve the following inequality and equality for x :

- (a) $(x - 5)^3(x + 1) \geq 0$,
- (b) $|x| = x^3 + 2x^2 - 3x$.

Exercise 3: Write each of the following sets as a union of intervals:

$$U = \left\{ x \in \mathbb{R} : ((x - 4)^2 - 10)^2 \geq 36 \right\}, V = \{ x \in \mathbb{R} : |2x + 6| + |2x - 6| - |x + 1| - |x - 1| > 8 \}.$$

Exercise 4: Evaluate the following sums:

$$(a) \sum_{n=7}^{42} \left(\frac{1}{3}\right)^n, \quad (b) \sum_{m=-1}^8 (n + 1)^3 \text{ for } n \in \mathbb{N}, \quad (c) \sum_{\mu=0}^1 \sum_{\nu=2}^4 \frac{1}{\mu + \nu^2}.$$

Exercise 5:

- (a) Change the indices of the summations in the following expression so that the expression may be written with only one summation sign.

$$\sum_{k=2}^{23} (k - 1)^2 + \sum_{\ell=-2}^{19} 2(\ell + 3) + \sum_{m=10}^{31} 1.$$

Then evaluate the sum with the help of some results from the lecture.

- (b) Prove the following identity for all $a, b \in \mathbb{R}$ and $n \in \mathbb{N}$ by means of changing indices:

$$(a - b) \sum_{k=0}^n a^k b^{n-k} = a^{n+1} - b^{n+1}.$$

Due date: Your written solutions are due at 14:00 on Tuesday, **October 30, 2018**.
 Please submit them in the beginning of the problem class.

Problem Session: 14:00 Tuesday, October 23, 2018

Website: For detailed information regarding this course visit the following web page: