Karlsruhe Institute of Technology
Institute for Algebra und Geometry
Prof. Axenovich Ph.D.

6	7	8	9	10	$\sum$

Karlsruhe, October 26, 2018

Name: ....

Exercise Sheet No. 2
Advanced Mathematics I

Exercise 6:

(a) Let $n, m, r \in \mathbb{N}$ and $n \geq m \geq r \geq 0$. Prove the following equality about binomial coefficients.

$$\binom{n}{m} \cdot \binom{m}{r} = \binom{n}{r} \cdot \binom{n-r}{m-r}.$$

(b) Evaluate the following summations:

(i)

$$\sum_{k=0}^{5} \binom{5}{k},$$

(ii)

$$\sum_{n=3} 5 \frac{\binom{n}{3}}{n!}.$$

Exercise 7:

(a) Prove that the sum of three consecutive natural numbers is always divisible by 3.

(b) Prove by contradiction that for each prime p, $\sqrt{p}$ is irrational (recall that 1 is not prime).

Exercise 8: Prove the following equality by induction:

$$\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

Exercise 9: Prove the following identities by induction:

(a)

$$\sum_{k=0}^{n} (2k+1) = (n+1)^2,$$

(b)

$$\sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)} = \frac{n}{2n+1}.$$

Exercise 10: Prove the following identity for $n \in \mathbb{N}$:

$$\sum_{k=1}^{n} \frac{(2k)! - 2 \cdot (2k-2)!}{2^k} = \frac{(2n)!}{2^n} - 1.$$

Due date: Your written solutions are due at 14:00 on Tuesday, **6 November, 2018**.
Please submit them at the beginning of the problem session
or in the box in J101 (note the box will be emptied before the problem session).

Problem Session: 14:00 Tuesday, November 6, 2018

Website: For detailed information regarding this course visit the following web page:

`http://www.math.kit.edu/iag6/edu/am12018w/en`