

11	12	13	14	15	$\Sigma$

### Exercise Sheet No. 3 Advanced Mathematics I

**Exercise 11:** Consider the complex numbers  $z_1 = 1 + i, z_2 = 2 - 3i, z_3 = \sqrt{3} + i$ .

- (a) Find the real and imaginary part of the numbers  $\bar{z}_j - z_j, z_j \bar{z}_j, \frac{1}{z_j}, z_j - \bar{z}_j$  and  $|z_j|$  for  $j = 1, 2$ .
- (b) Find a representation of  $z_3$  in polar coordinates  $(r, \phi)$ .

**Exercise 12:** Let  $z \in \mathbb{C} \setminus \{-i\}$  and define  $w$  as follows:

$$w = \frac{(\overline{2-i})z - 1 + 2i}{z + i}.$$

Compute the the real and imaginary part of  $w$  as a function of  $z$ , and also express  $w$  in polar form.

**Exercise 13:** Sketch the following regions of  $\mathbb{C}$ :

$$\begin{aligned} K &= \{z \in \mathbb{C} : |2z - 4i|^2 = 16\}, \\ R &= \{z \in \mathbb{C} : 1 \leq |z - 3 + 4i| \leq 3\}, \\ G &= \{z \in \mathbb{C} : |z - 1 + i| = |z - 2 - i|\}, \\ H &= \{z \in \mathbb{C} : \operatorname{Re}(z(1 - i)) \geq 0\}. \end{aligned}$$

**Exercise 14:** Solve the following quadratic equations over the complex numbers.

- (a) 
$$z^2 + (-10 + 4i)z + 70 - 20i = 0,$$
- (b) 
$$z^2 + 6z - 3 + i(4z + 6) = 0.$$

**Exercise 15:** Show the following by induction (Note:  $i = \sqrt{-1}$ ):

$$\sum_{k=0}^{2n} i^k k = \begin{cases} n(1 - i) & \text{when } n \text{ is even} \\ -(n + 1) + ni & \text{when } n \text{ is odd.} \end{cases}$$

**Due date:** Your written solutions are due at 14:00 on Tuesday, **13 November, 2018**.  
 Please submit them at the beginning of the problem session  
 or in the box in J101 (note the box will be emptied before the problem session).

**Website:** For detailed information regarding this course visit the following web page: