Karlsruhe Institute of Technology
Institute for Algebra und Geometry
Prof. Axenovich Ph.D.

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Karlsruhe, Jan 10, 2019

Name: .....

Exercise Sheet No. 11
Advanced Mathematics I

Exercise 51:

(a) Find the derivatives of

(i) $f_1(x) = x^x, \quad x \in \mathbb{R}_{>0},$

(ii) $f_2(x) = (\sqrt{x}+1)\left(\frac{1}{\sqrt{x}} - 1\right), \quad x \in \mathbb{R}_{>0},$

(iii) $f_3(x) = \frac{\sin x}{\sin x + \cos x}, \quad x \in \left[0, \frac{\pi}{2}\right]$

(iv) $f_4(x) = e^{(\sin x)^2} + e^{\sin(x^2)} + \left(e^{\sin x}\right)^2, \quad x \in \mathbb{R}.$

(b) Show that the function defined by $f(x) = x^a \sin(\frac{1}{x})$ for $x > 0$ and $f(0) = 0$ where $x \in \mathbb{R}_{\geq 0}$ is continuously differentiable when $a = 3$ and not differentiable when $a = 1$.

Exercise 52: Consider the function

$$f(x) = \begin{cases} e^x, & \text{if } x \leq 0, \\ \cos(x) + x, & \text{if } x > 0. \end{cases}$$

(a) Show that f is continuously differentiable at $x \neq 0$ arbitrarily many times.

(b) Show that f is (once) continuously differentiable at $x = 0$.

(c) Is f twice continuously differentiable at $x = 0$?

Exercise 53:

(a) Find all $x \in \mathbb{R}$, for which the function $f(x) = \sinh(x)$ is strongly monotone increasing (and thus has an inverse).

(b) Use the expression $\sinh(x) = \frac{1}{2}(e^x - e^{-x})$, to find the inverse function $f^{-1}(x) = \operatorname{Arcsinh}(x)$ and determine $(f^{-1})'(x)$.

Exercise 54:

Consider the function $f\colon \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \frac{x^2}{\sqrt{1+x^2}}.$$

Find the tangent line to f at the points $x_0 = 0, 1$ and $\sqrt{2}$. Show that the slope of the function $f(x)$ tends to 1 as $x \to \infty$.

Exercise 55:

Prove that for $x \geq e^2$ that the inequality

$$\sqrt{x} > \ln(x)$$

holds.

Hint: Consider the function $f(x) = \sqrt{x} - \ln x$.

Due date: Your written solutions are due at 14:00 on Tuesday, **22 January, 2019**.
Please submit them at the beginning of the problem session.

Website: For detailed information regarding this course visit the following web page:

`http://www.math.kit.edu/iag6/edu/am12018w/en`