Karlsruhe Institute of Technology
Institute for Algebra und Geometry
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56	57	58	59	60	$\sum$

Karlsruhe, Jan 17, 2019

Name: ....

Exercise Sheet No. 12
Advanced Mathematics I

Exercise 56: Compute the following limits:

$$\text{(a)} \quad \lim_{x\to 0}\left(\frac{1}{x^3+ax^2+x}-\frac{1}{\sin x}\right), \qquad \text{(b)} \quad \lim_{x\to 0}\left(\frac{1}{x}\right)^{\tan x}, \qquad \text{(c)} \quad \lim_{x\to 0}(\cos x)^{1/x^2}.$$

The coefficient $a \in \mathbb{R}$ in part (a) is a constant.

Exercise 57:

Determine the constant $c \in \mathbb{R}$ such that the function $f(x) = \begin{cases} c, & x = 1, \\ \frac{2^{\ln x}-x}{\ln x}, & x \neq 1, \end{cases}$ on $\mathbb{R}_{>0}$ is continuous.

Exercise 58: Calculate all the derivatives $f^{(n)}$, $n = 0, 1, 2, \dots$ of the function f and give the Taylor series for f with center of expansion $x_0 = 0$. Where does the series in part (a) converge?

$$\text{(a)} \quad f(x) = \cosh\frac{x}{2}, \quad x \in \mathbb{R}, \qquad \text{(b)} \quad f(x) = \sqrt{1+x}, \quad |x| \le 1.$$

Hint for (b): The derivatives of f have the following form: $f^{(k)}(x) = -(-1)^k \frac{(2k-2)!}{2^{2k-1}(k-1)!}(1+x)^{-\frac{2k-1}{2}}$.

Exercise 59:

(a) Determine the Taylor formula for $m = 2$ about the point $x_0 = 8$ for the function $f(x) = x^{2/3}$, $x \ge 1$, find an expression for the Lagrange form of the remainder.

(b) For natural numbers n and real x, $1 + x > 0$, show using the Taylor formula that

$$(1+x)^n \ge 1 + nx.$$

Exercise 60:

(a) Define the function $f\colon \mathbb{R} \to \mathbb{R}$ by $f(x) = 1$ for $x \ge 0$ and $f(x) = -1$ for $x < 0$. Show that the function $F(x) = \int_0^x f(t)dt$ is not a primitive (antiderivative) of f.

(b) Given $g\colon \mathbb{R}_{>0} \to \mathbb{R}$ defined by $g(x) = \int_{2\pi}^x \frac{\sin(t)}{t}dt$. Find the Taylor polynomial $p_2(x)$ of degree 2 about $x_0 = 2\pi$.

Due date: Your written solutions are due at 14:00 on Tuesday, **29 January, 2019**.
Please submit them at the beginning of the problem session.

Website: For detailed information regarding this course visit the following web page:

http://www.math.kit.edu/iag6/edu/am12018w/en