Karlsruhe Institute of Technology
Institute for Algebra und Geometry
Prof. Axenovich Ph.D.

61	62	63	64	65	$\sum$

Karlsruhe, Jan 17, 2019

Name: ....

Exercise Sheet No. 13
Advanced Mathematics I

Exercise 61:

Use integration by parts in order to determine the following integrals

a) $\displaystyle\int x\sin(2x)\,dx,$ b) $\displaystyle\int_0^{\pi/4} \frac{x}{\cos^2(x)}\,dx.$

Exercise 62:

Find antiderivatives of the functions in (a),(b) and (c). Find the derivative of the function in (d).

(a) $f\colon (1,\infty)\to\mathbb{R},$ $\displaystyle f(x) = \frac{1}{x\ln(x)\ln(\ln x)}$

(b) $g\colon (0,\infty)\to\mathbb{R},$ $\displaystyle g(x) = \left(\frac{\ln x}{x}\right)^2$

(c) $h\colon \mathbb{R}\to\mathbb{R},$ $\displaystyle h(x) = \frac{xe^{\arctan x}}{(1+x^2)^{\frac{3}{2}}}$

(d) $k\colon \mathbb{R}\to\mathbb{R},$ $k(x) = \int_{\cos(x^3)}^{x^2} t^4e^t dt.$

Exercise 63:

Find solutions for the initial value problems:

(a) $u' = e^{-u}\cos x,$ $u(0)=1,$ (b) $uu' + (1+u^2)\sin x = 0,$ $u(0)=1.$

Exercise 64:

Solve the following differential equations:

(a) $y'(x) - e^{-x} + y(x) - xy'(x) = xy(x),$ $y(0)=1$

(b) $x - y^2(x) + 2x\,y(x)\,y'(x) = 0$ for $x>0,$ $y(1)=1.$ *Hint:* Substitute $z(x)=y^2(x).$

Exercise 65: Solve the initial value problem

$$y^3(x) - x^2 + xy^2(x)y'(x) = 0, \qquad y(1)=1.$$

Due date: Your written solutions are due at 14:00 on Tuesday, **5 February, 2019**.
Please submit them at the beginning of the problem session.

Website: For detailed information regarding this course visit the following web page:

`http://www.math.kit.edu/iag6/edu/am12018w/en`