Karlsruhe Institute of Technology
Institute for Algebra und Geometry
Prof. Axenovich Ph.D.

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Karlsruhe, December 17, 2018

Name: ....

Exercise Sheet No. 9
Advanced Mathematics I

Exercise 41: For each of the following power series find all $z \in \mathbb{C}$ where the series converges

$$\text{(a)} \quad \left(\sum_{n=0}^{\infty} \frac{z^n}{2(n!)}\right), \quad \text{(b)} \quad \left(\sum_{n=1}^{\infty} \frac{(z-1)^{2n}}{\left(1+\frac{1}{n}\right)^n}\right), \quad \text{(c)} \quad \left(\sum_{n=1}^{\infty} n^n (z+2)^n\right).$$

Exercise 42: For which $x \in$ does the power series

$$\sum_{n=1}^{\infty} (-2)^n \frac{n^2+2}{n^3+n} x^{3n}$$

converge?

Attention: The solution set is an interval. Does it contain its boundary points?

Exercise 43: Expand the function $f : \mathbb{R} \setminus \{1\} \to \mathbb{R}$, $f(x) = \dfrac{e^x}{1-x}$ as a power series $(\sum_{n=0}^{\infty} a_n x^n)$ centered at 0.

(a) Show that $a_n = \displaystyle\sum_{k=0}^{n} \frac{1}{k!}$ for all $n \in \mathbb{N}_0$.

(b) Find all $x \in \mathbb{R}$ where the power series converges.

Exercise 44: Consider the functions $\dfrac{1}{z+1}$, $z \in \mathbb{C} \setminus \{-1\}$, and $\dfrac{1}{z+2}$, $z \in \mathbb{C} \setminus \{-2\}$.

(a) Expand both functions as power series centered at $z_0 = 1$ and find the radii of convergence.

(b) Expand the function $\left(\frac{1}{z+1}\right) \cdot \left(\frac{1}{z+2}\right)$ as a power series centered at $z_0 = 1$ using the Cauchy product. Find a closed form representation of the coefficients!

(c) Expand the function $\left(\frac{1}{z+1}\right) \cdot \left(\frac{1}{z+2}\right)$ as a power series centered at $z_0 = 1$ using partial fraction decomposition.

Hint: This means find numbers a, b such that $\left(\frac{1}{z+1}\right) \cdot \left(\frac{1}{z+2}\right) = \left(\frac{a}{z+1}\right) + \left(\frac{b}{z+2}\right)$ for all z and proceed.

(d) Find all $z \in \mathbb{C}$ where the Cauchy product from b) converges.

Exercise 45:

(a) Give the exponential representation $z = re^{i\varphi}$, $\varphi \in (-\pi, \pi]$, of the following complex numbers

$$\text{(i)} \quad -1+i, \qquad \text{(ii)} \quad (1-i)e^{-i\pi}, \qquad \text{(iii)} \quad \frac{3+4i}{1-i}, \qquad \text{(iv)} \quad \frac{1-i}{i+2}.$$

(b) Show that the following identities apply for all $z \in \mathbb{C}$:

$$\text{(i)} \quad \cos(iz) = \cosh z \text{ and } \cos z = \cosh(iz), \qquad \text{(ii)} \cos(\overline{z}) = \overline{\cos(z)}.$$

Due date: Your written solutions are due at 14:00 on Tuesday, **8 January, 2019**. Please submit them at the beginning of the problem session.

Website: For detailed information regarding this course visit the following web page:

`http://www.math.kit.edu/iag6/edu/am12018w/en`