Karlsruhe Institute of Technology
Institute for Algebra und Geometry
Prof. Axenovich Ph.D.

36	37	38	39	40	$\sum$

Karlsruhe, December 6, 2018

Name: .....

Exercise Sheet No. 8
Advanced Mathematics I

Exercise 36:

Given the series $\left(\sum_{n=1}^{\infty} a_n\right)$ and $\left(\sum_{n=m}^{\infty} b_n\right)$, where $a_n = \frac{1}{n(n+1)}$, $b_n = \frac{2}{3^n}$ for $n \in \mathbb{N}$ and $m \in \mathbb{N}$.

(a) Determine the first partial sums of the series $\left(\sum_{n=1}^{\infty} a_n\right)$ and $\left(\sum_{n=m}^{\infty} b_n\right)$.

(b) Show that $\left(\sum_{n=1}^{\infty} a_n\right)$ converges and find the limit $\sum_{n=1}^{\infty} a_n$.

(c) Show that $\left(\sum_{n=m}^{\infty} b_n\right)$ converges and find the limit $\sum_{n=m}^{\infty} b_n$.

Exercise 37: Determine if the following series are convergent or absolutely convergen

$$\text{(a)} \left(\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k}{2^k}\right), \qquad \text{(b)} \left(\sum_{k=1}^{\infty}(-1)^{k+1}\frac{\sqrt{k+\frac{1}{k}}}{k}\right)$$

In the case of convergence find an index N after which the partial sums s_n for $n \geq N$ differ by no more than 10^{-2} from the limit.

Exercise 38:

Determine whether the following sequences converge using the quotient or root criteria.

$$\text{(a)} \left(\sum_{k=1}^{\infty}\left(\frac{10}{9}+\frac{1}{k}\right)^k\right), \quad \text{(b)} \left(\sum_{k=1}^{\infty}\frac{(k+1)!}{(2k)!}\right), \quad \text{(c)} \left(\sum_{k=1}^{\infty}2^{-\frac{k^2+1}{k+1}}\right), \quad \text{(d)} \left(\sum_{k=1}^{\infty}\frac{k!}{k^k}\right).$$

Exercise 39: Show that each of the following series either converges or diverges, by constructing a larger convergent series, or smaller divergent series, respectively, and then using the comparison test.

$$\text{a)} \left(\sum_{k=8}^{\infty}\frac{2k+3\sqrt{k}+3\sqrt[3]{k}}{k^3-k}\right), \quad \text{b)} \left(\sum_{k=0}^{\infty}\frac{\sqrt[4]{k+1}-\sqrt[4]{k}}{\sqrt[3]{k+2}}\right), \quad \text{c)} \left(\sum_{k=0}^{\infty}\frac{\sqrt[4]{k+1}-\sqrt[4]{k}}{\sqrt[4]{k+3}}\right).$$

Exercise 40:

(a) For which $q \in \mathbb{R}$ does the following sequence converge: $\sum_{n=0}^{\infty}(n+1)q^n$?

(b) Examine the convergence of the following sequences and determine the limit if it exists:

$$\text{(i)} \left(\sum_{n=0}^{\infty}\left(\frac{3+4i}{6}\right)^n\right), \qquad \text{(ii)} \left(\sum_{k=1}^{\infty}\frac{1}{\sqrt[k]{2}}\right).$$

Due date: Your written solutions are due at 14:00 on Tuesday, **18 December, 2018**.
Please submit them at the beginning of the problem session.

Website: For detailed information regarding this course visit the following web page:

`http://www.math.kit.edu/iag6/edu/am12018w/en`