Karlsruhe Institute of Technology
Institute for Algebra und Geometry
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21	22	23	24	25	$\sum$

Karlsruhe, November 16, 2018

Name: ....

Exercise Sheet No. 5
Advanced Mathematics I

Exercise 21: Determine all accumulation points of the following sequences

(a) $a_n := \frac{1}{n} + 2(-1)^n, \quad n \in \mathbb{N},$ (b) $b_n := \left(\frac{5n+7}{n}\right) i^n, \quad n \in \mathbb{N}.$

Exercise 22: We discuss the recursively defined sequence

$$a_1 = b\,, \qquad a_{k+1} = \frac{|a_k|}{2a_k - 1}\,,\ k \in \mathbb{N}$$

for two initial values $b = -\frac{1}{4}$, and $b = \frac{1}{4}$.

(a) Assume that for fixed b the sequence (a_k) converges. What are the candidates for the limit?

(b) Determine for which initial value $b = -\frac{1}{4}$ or $b = \frac{1}{4}$ the sequence is monotone.

(c) Determine for which initial value the sequence is bounded.

(d) Justify, for both initial values $b = -\frac{1}{4}$ or $b = \frac{1}{4}$, whether the sequence is converging or not. In case of convergence determine the limit.

Exercise 23: Let the sequence $(c_n)_{n=1}^{\infty}$ in $\mathbb{C}$ be recursively defined by

$$c_1 = 1 + 2i \qquad \text{and} \qquad c_{n+1} = \frac{2\mathrm{Re}(c_n)\mathrm{Im}(c_n)}{\mathrm{Re}(c_n) + \mathrm{Im}(c_n)} + i\sqrt{\mathrm{Re}(c_n)\mathrm{Im}(c_n)} \quad \text{for all } n \in \mathbb{N}.$$

(a) Show that $\mathrm{Re}(c_1) \le \mathrm{Re}(c_2) \le \ldots \le \mathrm{Re}(c_n) \le \mathrm{Im}(c_n) \le \ldots \le \mathrm{Im}(c_2) \le \mathrm{Im}(c_1)$ for all $n \in \mathbb{N}$.

(b) Show that (c_n) converges and that $\mathrm{Re}(c) = \mathrm{Im}(c)$ holds for the limit c of the sequence.
Note: You don't have to compute c.

Exercise 24: Which of the following assertions are true? Give a counter example for each incorrect assertion.

(a) If a sequence is monotone and bounded, then it converges.

(b) If a sequence converges, then it is monotone and bounded.

(c) If a sequence is not bounded, then it is not convergent.

(d) If a sequence is not monotonic, then it is not convergent.

(e) If a sequence has exactly one accumulation point, then it converges.

(f) If a sequence converges, then it has exactly one accumulation point.

Exercise 25:

Determine the image of the set $M := \{z \in \mathbb{C} : |z| = 1\}$ under the mapping

$$f : \begin{cases} \mathbb{C} \setminus \{-3/2 + i/2\} \longrightarrow \mathbb{C}, \\ z \mapsto \dfrac{-2}{2z + 3 - i}. \end{cases}$$

Sketch the sets M and $f(M)$.

Due date: Your written solutions are due at 14:00 on Tuesday, **27 November, 2018**.
Please submit them at the beginning of the problem session
or in the box in J101 (note the box will be emptied before the problem session).

Website: For detailed information regarding this course visit the following web page:

`http://www.math.kit.edu/iag6/edu/am12018w/en`