Karlsruhe Institute of Technology
Institute for Algebra und Geometry
Prof. Axenovich Ph.D.

26	27	28	29	30	$\sum$

Karlsruhe, November 23, 2018

Name: ....

Exercise Sheet No. 6
Advanced Mathematics I

Exercise 26: Consider the polynomial $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) := \frac{1}{8}x^3 + \frac{3}{8}x^2 - \frac{9}{8}x + \frac{5}{8}$.

(a) Express $f(x)$ in terms of $x-1$ and $x+3$. Use this representation to discuss the behavior of f on the intervals $[-5, \infty)$ and $(-\infty, 3]$. Do not use the derivative f' for this discussion.

(b) Use a sketch of f to find intervals on which f has an inverse function. Also sketch the inverse on these intervals.

Exercise 27: Decide whether the functions are injective, surjective or bijective. If the function is bijective, then find the inverse. Justify your answer.

(a) $f\colon \mathbb{R} \to \mathbb{R}_{\geq 0}$, $f(x) = |x|$

(b) $g\colon \mathbb{N} \to \mathbb{N}$, $g(n) = n + 1$

(c) $h\colon \mathbb{N}_0 \to \mathbb{Z}$, $h(n) = \begin{cases} \frac{n}{2}, & \text{for } n \text{ even} \\ -\frac{n+1}{2}, & \text{for } n \text{ odd} \end{cases}$

Exercise 28:

Using the sequence definition of continuity, find $w \in \mathbb{R}$ so that the function

$$f(x) = \frac{x\sqrt{x} - 1}{\sqrt{x} - 1}, \qquad x > 0,\ x \neq 1,$$

is continuous if we set $f(1) := w$.

Exercise 29:

The function $f : \mathbb{R}\backslash\{-2\} \to \mathbb{R}$ is defined as

$$f(x) = \begin{cases} \dfrac{5x \cdot |x-3|}{x^2 - x - 6} & , \quad x \in \mathbb{R}\backslash\{-2, 1, 3\}\ , \\ y_1 & , \quad x = 1\ , \\ y_2 & , \quad x = 3\ . \end{cases}$$

Is it possible for f to be continuous at $x = 1$ and $x = 3$ with a suitable choice of y_1, y_2? Give the appropriate values, or show that none exist.

Exercise 30:

Prove the following functions are Lipschitz-continuous and find the respective Lipschitz constant.

(a) $f(x) = \sqrt{1 + 4x}$, $D = [0, 4)$,

(b) $f(x) = x^2 + 4x - 1$, $D = (-4, 3)$,

(c) $f(x) = \sqrt{2x^2 + 1}$, $D = [-2, 1]$.

Due date: Your written solutions are due at 14:00 on Tuesday, **4 December, 2018**. Please submit them at the beginning of the problem session.

Website: For detailed information regarding this course visit the following web page:

`http://www.math.kit.edu/iag6/edu/am12018w/en`