Problem Sheet 3

Due date: **May 15, 2017, 15:30**. Discussion of solutions: May 15, 2017.

Problem 7. **6 points**

You are organizing a dinner with n guests and your venue offers tables with exactly n seats in total. Let t_i denote the number of tables with i seats, $i \in [n]$. So $\sum_{i=1}^{n} it_i = n$ and $0 \leq t_i \leq n$ for each $i \in [n]$. How many different assignments of the guests to the tables (but not to particular seats or orderings around a table) are there if

(a) all the tables are distinguishable?

(b) tables of the same size are not distinguishable?

(c) some k very important guests each need a table on their own? (Assume $t_1 \geq k$ and tables of the same size are still not distinguishable.)

Problem 8. **6 points**

Consider the equation $x_1 + x_2 + x_3 + x_4 + x_5 = 42$ for $x_i \in \mathbb{Z}$.

(a) How many solutions does the equation have, if $x_i \geq 0$ for each $i \in [5]$?

(b) How many solutions does the equation have, if $x_1 > 0$, $x_2, x_3, x_4, x_5 \geq 0$ and $x_3 \leq 10$?

(c) How many solutions exist, if $0 \leq x_i$, for each $i \in [5]$, and the equality-sign is replaced by "$\leq$"?

(d) How many solutions exist, if $0 \leq x_i \leq 42$, for each $i \in [5]$, and the equality-sign is replaced by "$>$"?

Do not compute binomial coefficients explicitly.

Problem 9. **6 points**

Ten taxis are waiting in front of your venue, although only seven guests are about to leave. In how many different ways may these guests share their rides? Assume that each taxi may take all the guests and all guests enter the taxis at the same time. Compute each number in the following table explicitly.

	no two guests share a taxi	taxis may be shared
taxis and guests labeled	(a)	(b)
taxis unlabeled, guests labeled	(c)	(d)
taxis labeled, guests unlabeled	(e)	(f)
taxis and guests unlabeled	(g)	(h)

Maria Axenovich
Torsten Ueckerdt
Jonathan Rollin