Problem Sheet 8

Due date: **June 19, 2017, 15:30**. Discussion of solutions: June 19, 2017.

Problem 22. **6 points**

Someone offers you the following bet for a fixed constant $\alpha \in [0, 1]$:

- First, you invest some number n of coins between 0 and 100 of your choice.
- Then a random game proceeds in rounds, where in each round with probability α you lose one coin and with probability $1 - \alpha$ you gain one coin.
- When you reach 0 coins, you lose the game and also the n coins you invested.
- When you reach 100 coins, you win the game and also the $100 - n$ coins you gained.

Depending on α, what is the smallest number n of coins you have to invest in order to win the game with probability at least 1/2?

Hint: Define P_n to be the probability of winning the game with n coins, find a recurrence relation for $(P_n)_{n\in\mathbb{N}}$ and solve it.

Problem 23. **6 points**

Solve the following interdependent recurrence relations:

$$\begin{aligned} A_0 &= 1, \quad A_n = 3A_{n-1} + 2B_{n-1} + 3^n, \quad n \geq 1 \\ B_0 &= 2, \quad B_n = A_{n-1} + 2B_{n-1}, \quad n \geq 1 \end{aligned}$$

Hint: Consider the ordinary generating function for $(A_n)_{n\in\mathbb{N}}$ and $(B_n)_{n\in\mathbb{N}}$.

Problem 24. **6 points**

Consider the recurrence relation

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + s^n \quad n \geq 2$$

with $c_2, s \neq 0$.

(a) Prove that if s is a root of the characteristic polynomial of multiplicity 1, then the recurrence relation is solved by $a_n = A \cdot ns^n$ for some constant A.

(b) Prove that if s is a root of the characteristic polynomial of multiplicity 2, then the recurrence relation is solved by $a_n = A \cdot n^2 s^n$ for some constant A.

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