CONSTRUCTIVE LOWER BOUND FOR SYMMETRIC RAMSEY NUMBERS BY FRANKL AND WILSON

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Let $\mathcal{F}$ be a family of $k$-element subsets of an $n$-element set. By the result of Ray-Chaudhuri and Wilson [2],

$${\left|\{F \cap F': F, F' \in \mathcal{F}\}\right|} \leq s,$$  \hspace{1cm} (1)

then $\left|\mathcal{F}\right| \leq \binom{n}{s}$.

By the result of Frankl and Wilson [1],

if $|F \cap F'| \not\equiv k \pmod{q}$ for a prime power $q$ then $\left|\mathcal{F}\right| \leq \binom{n}{q-1}$. \hspace{1cm} (2)

**Theorem 1** (Frankl and Wilson [1]). When $k$ is sufficiently large, $r(k) \geq \exp(\log^2 k/20 \log \log k)$.

**Proof.** Let $V(G) = \binom{X}{q-1}$, where $|X| = q^3$ and $q$ is a sufficiently large prime power. Let

$E(G) = \{\{F, F'\} : |F \cap F'| \not\equiv -1 \pmod{q}\}$.

If $F_1, \ldots, F_m$ form a complete graph, then $m \leq \binom{q^3}{q-1}$ by (2). If $F_1, \ldots, F_m$ form an independent set, then the pairwise intersections have sizes $q - 1, 2q - 1, \ldots, q^2 - q - 1$, so $m \leq \binom{q^3}{q-1}$ by (1). So, $G$ has no clique or co-clique on $k$ vertices, where

$|V(G)| = \binom{q^3}{q^2 - 1}$ and $k = \binom{q^3}{q-1}$.

Using the bounds $\binom{a}{m} \leq \binom{n}{m} \leq n^m$, we have that

$q^3 \leq k \leq q^{3q}$ and $|V(G)| \geq q^{q^2/2}$.

So $q \log q \leq \log k \leq 3q \log q$ and thus $\log k/3 \log q \leq q \leq \log k/\log q$. Therefore $\log q \leq \log \log k - \log \log q \leq \log k/3 \log \log k$. Therefore

$|V(G)| \geq \exp\left(1 + \log^2 k/18(\log \log k)^2\right)$

$= \exp\left(\log^2 k \left(\log \log k - \log 3 - \log \log \log k\right)/18(\log \log k)^2\right)$

$\geq \exp(\log^2 k/20 \log \log k)$.

Note that this gives that $r(k) \geq k^{c\sqrt{\log k}}$, i.e., this bound is greater than any power of $k$ but smaller than exponential. The best constructive bound up to date is due to Barak et al., [3], $r(k) \geq \exp((1 + o(1)) \log (2 + a)k)$, for a positive constant $a$.

**References**


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