Problem sheet 2

Due date: **November 6, 12:00 am.** Discussion of solutions: November 8.

**Problem 5.** 5 points
Let $G$ be a nonempty graph with minimum degree at least two. Show that there is a connected graph having the same degree sequence as $G$.

**Problem 6.** 5 points
Let $T$ be a tree with an even number of vertices. Show that $T$ has exactly one spanning subgraph in which every vertex has odd degree.

**Problem 7.** 5 points
For any graph $G$ let $\pi(G)$ denote the minimum number of walks in $G$ so that every edge of $G$ appears once in exactly one walk and does not appear in other walks. Find an expression/formula for $\pi(G)$.

**Problem 8.** 5 points
A *permutation matrix* is a matrix of zeros and ones such that each row and each column contains exactly one 1.

Show that a square matrix $A$ with nonnegative integer entries is a sum of $k$ permutation matrices if and only if the sum of elements in each row and in each column of $A$ is $k$.

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**Open Problem.**
Let $G$ be a $k$-regular graph with $k$ odd. Is there always a set $S$ of paths so that every edge of $G$ is in exactly one path and every vertex of $G$ is the endpoint of exactly one path in $S$?