Problem sheet 14

Due date: **February 12, 12:00 am.**

Discussion of solutions: February 7.

(农家乐：Please prepare solutions for **at most three** problems.)

**Problem 53.** 5 points
Show that every $\mathbb{R}$-flow $f$ on a graph $G$ decomposes into cycles, i.e., that there exists $\mathbb{R}$-flows $f_1, \ldots, f_k$ in $G$ such that $f = \sum_{i=1}^{k} f_i$ and for $i = 1, \ldots, k$ the set of edges with non-zero flow value in $f_i$ is a cycle.

**Problem 54.** 5 points
Let $H$ be a group, $G$ be a connected graph and $T$ a spanning tree of $G$. Prove that every $H$-flow $f$ on $G$ is uniquely determined by its values on the edges not in $T$.

**Problem 55.** 5 points
Let $T$ be the infinite complete $k$-ary tree with root $r$. For some fixed $0 < p < 1$ every edge of $T$ is deleted independently with probability $p$. Determine the expected number of vertices in the component containing $r$.

**Problem 56.** 5 points
A tournament is a set of $n$ teams and one match between any two teams. Assume that every match has a winner, there is no draw. Is it possible that in some tournament for every triple of teams there exists a fourth team that wins against each team in the triple?

**Open Problem.**
Prove or disprove that a random graph $G(2^d, \frac{1}{2})$ almost surely contains a spanning $d$-dimensional cube.