Convex-Arc Drawings of Pseudolines*

David Eppstein[1], Mereke van Garderen[2],
Bettina Speckmann[2], and Torsten Ueckerdt[3]

[1] Computer Science Dept., University of California, Irvine, USA
eppstein@uci.edu
[2] Dept. of Mathematics and Computer Science, TU Eindhoven, the Netherlands
m.v.garderen@student.tue.nl, speckman@win.tue.nl
[3] Dept. of Mathematics, KIT, Germany
torsten.ueckerdt@kit.edu

Introduction. A *pseudoline* is formed from a line by stretching the plane without tearing: it is the image of a line under a homeomorphism of the plane [13]. In *arrangements* of pseudolines, pairs of pseudolines intersect at most once and cross at their intersections. Pseudoline arrangements can be used to model sorting networks [1], tilings of convex polygons by rhombi [4], and graphs that have distance-preserving embeddings into hypercubes [6]. They are also closely related to oriented matroids [11]. We consider here the visualization of arrangements using well-shaped curves.

Primarily, we study *weak outerplanar* pseudoline arrangements. An arrangement is *weak* if it does not necessarily have a crossing for every pair of pseudolines [12], and *outerplanar* if every crossing is part of an unbounded face of the arrangement. We show that these arrangements can be drawn with all curves convex, either as polygonal chains with at most two bends per pseudoline or as semicircles above a line. Arbitrary pseudolines can also be drawn as convex curves, but may require linearly many bends.

Related Work. Several results related to the visualization of pseudoline arrangements are known. In *wiring diagrams*, pseudolines are drawn on parallel horizontal lines, with crossings on short line segments that connect pairs of horizontal lines [10]. The graphs of arrangements have drawings in small grids [8] and the dual graphs of weak arrangements have drawings in which each bounded face is centrally symmetric [5]. The pseudoline arrangements in which each pseudoline is a translated quadrant can be used to visualize *learning spaces* representing the states of a human learner [7]. Researchers in graph drawing have also studied force-directed methods for schematizing systems of curves representing metro maps by replacing each curve by a spline; these curves are not necessarily pseudolines, but they typically have few crossings [9].

Results. Below we state our results for outerplanar and arbitrary arrangements.

Theorem 1. *Every weak outerplanar pseudoline arrangement may be represented by a set of chords of a circle.*

Corollary 1. *Every weak outerplanar pseudoline arrangement may be represented by lines in the hyperbolic plane, or by semicircles with endpoints on a common line.*

* Parts of this work originated at Dagstuhl seminar 13151, *Drawing Graphs and Maps with Curves*. D.E. was supported in part by the National Science Foundation under grants 0830403 and 1217322, and by the Office of Naval Research under MURI grant N00014-08-1-1015.

S. Wismath and A. Wolff (Eds.): GD 2013, LNCS 8242, pp. 522–523, 2013.
© Springer International Publishing Switzerland 2013

This result complements the fact that a weak arrangement with no 3-clique can always be represented by hyperbolic lines, regardless of outerplanarity [2].

Corollary 2. *Every weak outerplanar pseudoline arrangement may be represented by convex polygonal chains with only two bends.*

Theorem 2. *Every n-element pseudoline arrangement can be drawn with convex polylines, each of complexity at most n.*

For smooth curves composed of multiple circular arcs and straight line segments, Bekos et al. [3] defined the *curve complexity* to be the maximum number of arcs and segments in a single curve. By replacing each bend of the above result by a small circular arc, one obtains a smooth convex representation of the arrangement with curve complexity $O(n)$. We can show that these bounds are optimal.

Theorem 3. *There exist simple arrangements of n pseudolines that, when represented by polygonal chains require some pseudolines to have $\Omega(n)$ bends.*

Theorem 4. *There exist simple arrangements of n pseudolines that, when represented by smooth piecewise-circular curves require some curves to have $\Omega(n)$ arcs.*

References

1. Angel, O., Holroyd, A.E., Romik, D., Virág, B.: Random sorting networks. Advances in Mathematics 215(2), 839–868 (2007)
2. Bandelt, H.-J., Chepoi, V., Eppstein, D.: Combinatorics and geometry of finite and infinite squaregraphs. SIAM Journal of Discrete Mathematics 24(4), 1399–1440 (2010)
3. Bekos, M.A., Kaufmann, M., Kobourov, S.G., Symvonis, A.: Smooth orthogonal layouts. In: Didimo, W., Patrignani, M. (eds.) GD 2012. LNCS, vol. 7704, pp. 150–161. Springer, Heidelberg (2013)
4. da Silva, I.P.F.: On fillings of $2N$-gons with rhombi. Disc. Math. 111(1-3), 137–144 (1993)
5. Eppstein, D.: Algorithms for drawing media. In: Pach, J. (ed.) GD 2004. LNCS, vol. 3383, pp. 173–183. Springer, Heidelberg (2005)
6. Eppstein, D.: Cubic partial cubes from simplicial arrangements. Electronic Journal of Combinatorics 13(1), 79 (2006)
7. Eppstein, D.: Upright-quad drawing of *st*-planar learning spaces. Journal of Graph Algorithms and Applications 12(1), 51–72 (2008)
8. Eppstein, D.: Drawing arrangement graphs in small grids, or how to play Planarity. In: Wismath, S., Wolff, A. (eds.) GD 2013. LNCS, vol. 8242, pp. 436–447. Springer, Heidelberg (2013)
9. Fink, M., Haverkort, H., Nöllenburg, M., Roberts, M., Schuhmann, J., Wolff, A.: Drawing metro maps using bézier curves. In: Didimo, W., Patrignani, M. (eds.) GD 2012. LNCS, vol. 7704, pp. 463–474. Springer, Heidelberg (2013)
10. Goodman, J.E.: Proof of a conjecture of Burr, Grünbaum, and Sloane. Discrete Mathematics 32(1), 27–35 (1980)
11. Goodman, J.E.: Pseudoline arrangements. In: Handbook of Discrete and Computational Geometry, page 97 (2010)
12. Goodman, J.E., Pollack, R.: Allowable sequences and order types in discrete and computational geometry. In: Pach, J. (ed.) New Trends in Discrete and Computational Geometry. Algorithms and Combinatorics, vol. 10, pp. 103–134. Springer, Heidelberg (1993)
13. Shor, P.W.: Stretchability of pseudolines is NP-hard. In: Gritzmann, P., Sturmfels, B. (eds.) Applied Geometry and Discrete Mathematics: The Victor Klee Festschrift. DIMACS Series in Discrete Mathematics and Theoretical Computer Science, vol. 4, pp. 531–554. Amer. Math. Soc., Providence (1991)