Prof. Dr. Roman Sauer
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Summer term 2016
KIT Karlsruhe

Seminar: Selected Topics in Algebraic Topology

Zeit:	Mittwoch 15.45-17.15
Ort:	20.30 SR 3.61
Erster Termin:	20.04.2016

The program is structured into five thematic blocks. Speakers within one block are encouraged to discuss and plan their presentations together. During your preparation it might turn out to be necessary to distribute the material within one block a bit differently than indicated below.

Draw pictures whenever possible and helpful! We expect a blackboard presentation but you may use a projector for pictures.

Take the time planning seriously. Each presentation should take about 80 minutes (not 90 minutes!). It is important to decide in advance which details to present (because they are essential for understanding or providing context) or to drop (because they are mere technical details that do not enhance the understanding).

Please prepare a brief handout of your talk with the most important definitions and theorems (but no proofs) a week before your presentation. The handout will be uploaded and made available to all participants before your presentation.

Programm

Aspects of group homology – guided by Thompson's group F

This series is a bit more on the group theory side but its methods use homotopy theory. In the third talk also ℓ^2-Betti numbers show up. We learn about finiteness conditions of classifying spaces of groups. For that Brown's book [3] is an excellent background reference.

The first example of a torsionfree group with infinite cohomological dimension and property F_∞ was Thompson's group F. This famous group is also the focus of a famous open problem: Is Thompson's group F amenable or not? Hence this group is a natural guide through this block.

For the first talk some knowledge of geometric group theory is helpful but not strictly needed. The third talk assumes the course on ℓ^2-invariants by Holger Kammeyer.

Presentation 1 (27.04.16)**.** Explain the notions of type F_n and geometric dimension of a group. Relation to finite presentation and finite generation. Show that Thompson's group F has infinite geometric dimension. Introduce the fundamental Morse lemma 2.2.20 in [1].
Speaker: Xandra Boge
Literature: [1][3]

Presentation 2 (04.05.16)**.** Prove the Brown-Geoghegan theorem saying that Thompson's group F is of type F_∞ based on the beautiful proof [1, Theorem 2.2.21].
Speaker: Leonid Grau
Literature: [1][3]

Presentation 3 (11.05.16)**.** Provide a very short (!) survey about amenability. Take [6, Section 6.4.1] as a starting point. Show that the ℓ^2-Betti numbers of Thompson's group F vanish [6, Theorem 7.10].The latter result assumes knowledge of ℓ^2-Betti numbers which shall not be defined in the talk. However, also the non ℓ^2-audience should be able to grasp the relevance of the vanishing result for the amenability conjecture.
Speaker: Florian Kunick
Literature: [6]

Obstruction theory

This block deals with the extension problem of extending a map $A \to Y$ to $X \to Y$ for a relative CW-complex (X,A), and provides cohomological obstructions to a solution. After that we discuss geometric applications. Our main reference is Bredon's book but the speakers should also consult Hatcher's book [4, Section 4.3].

In addition to the courses on algebraic topology some basic notions from homotopy theory (like the notion of fibration) are needed which are covered in the beginning of the course on homotopy theory (and can be easily acquired independently).

Presentation 4 (18.05.16)**.** Prove the Moore-Postnikov decomposition of a fibration $Y \to B$ (Theorem 13.7 in [2]). If B is a point, then this is called the Postnikov-decomposition of Y which is kind of dual to a CW-decomposition.
Speaker: David Degen
Literature: [2, Section VII.13 up to Proposition 13.8]

Presentation 5 (25.05.16)**.** The main result is [2, Theorem 13.11] which develops the obstruction theory. Discuss all the corollaries.
Speaker: AG Topologie
Literature: [2, Section VII.13 starting from Proposition 13.8]

Presentation 6 (01.06.16)**.** Discuss as many geometric applications of the obstruction theory as possible. Especially important are Corollary 14.5 and 14.8.
Speaker: AG Topologie
Literature: [2, Section VII.14]

Steenrod operations - construction and applications

Singular cohomology has more structure than the cup product. It is a module over a non-commutative algebra, the Steenrod algebra. This extra structure has many applications, most notably to computations of homotopy groups. We follow Hatcher's account but one is encouraged to look at [7] as well.

Presentation 7 (08.06.16)**.** Introduce cohomology operations, Steenrod operations and their properties. Discuss Theorems 4L.2 and 4L.4 and more applications of your choice.

Speaker: Jan Kohlmüller
Literature: [4, Section 4L, p. 487–494]

Presentation 8 (15.06.16)**.** Examples 4L.6 and 4L.7. Discuss the Adem relations and the Steenrod algebra.
Speaker: Steven Grothnes
Literature: [4, p. 495–499]

Presentation 9 (22.06.16)**.** Explain the construction for the prime 2. Choose what to present in detail.
Speaker: Fabian Kellermann
Literature: [4, p. 501–509]

Presentation 10 (29.06.16)**.** Explain the construction for odd primes. Choose what to present in detail.
Speaker: AG Topologie
Literature: [4, p. 509–517]

Eilenberg-Mac Lane spaces and homotopy groups of spheres

This block builds on the previous one. In addition, one needs the Leray spectral sequence which is introduced in the course on homotopy theory. We learn how to calculate the cohomology of Eilenberg-Mac Lane spaces. Combined with the Postnikov decomposition from the series on obstruction theory, this can be applied to a computation of some homotopy groups of spheres in the last talk.

Presentation 11 (06.07.16)**.** Prove Serre's theorem 5.31. Skip the proof of Theorem 5.35. Theorem 5.36 and 5.37 need only to be stated.
Speaker: Jakob Albers
Literature: [5, p. 562–570]

Presentation 12 (13.07.16)**.** Prove Theorem 5.39 which computes some homotopy groups of spheres. The proof is based on Steenrod operations and spectral sequences.
Speaker: Jakob Albers
Literature: [5]

References

[1] M. Blank and W. Thumann, *Geometric group theory II*, 2014. Lecture notes, available under www.mathematik.uni-regensburg.de/blank/ggtIIss15/ggtii.pdf.

[2] G. E. Bredon, *Topology and geometry*, Graduate Texts in Mathematics, vol. 139, Springer-Verlag, New York, 1993.

[3] K. S. Brown, *Cohomology of groups*, Graduate Texts in Mathematics, vol. 87, Springer-Verlag, New York, 1994. Corrected reprint of the 1982 original.

[4] A. Hatcher, *Algebraic topology*, Cambridge University Press, Cambridge, 2002.

[5] ———, *Algebraic topology (Chapter 5).* available under: www.math.cornell.edu/ hatcher/AT/ATch5.pdf.

[6] W. Lück, *L^2-invariants: theory and applications to geometry and K-theory*, Vol. 44, Springer-Verlag, Berlin, 2002.

[7] R. E. Mosher and M. C. Tangora, *Cohomology operations and applications in homotopy theory*, Harper & Row, Publishers, New York-London, 1968.