Exercise 1 C

Let $H$ be a Hilbert space and $(u_n)_n \subset H$ be such that $u_n \rightharpoonup u$ weakly. Assume that

$$\limsup \|u_n\| \leq \|u\|.$$ 

Prove that $u_n \to u$ strongly without relying on Proposition 7.45.

Exercise 2 C

Let $H$ be a Hilbert space and $A$ be a closed subspace of $H$. Show that each vector $y \in H$ has a unique representation

$$y = a + b, \quad \text{where } a \in A \text{ and } b \in A^\perp.$$