Institut für Analysis
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Functional Analysis

3rd Exercise Sheet

* Exercise 7: Closed Subspaces

Show that the following subsets form closed linear subspaces of given normed spaces:

(1) Consider a complete normed space $(B(X), \|\cdot\|_\infty)$ of bounded functions on a metric space (X, d) with supremum norm and following sets:

(a) bounded continuous functions $C_b(X) = \{f : X \to \mathbb{K} | f$ is bounded and continuous$\}$,

(b) bounded uniformly continuous functions $BUC(X) = \{f : X \to \mathbb{K} | f$ is bounded and uniformly continuous$\}$.

(2) Consider a complete normed space $(l^\infty, \|\cdot\|_\infty)$ of bounded sequences in $\mathbb{K}$ with supremum norm and set of converging sequences $c = \{x = (x_j)_{j\in\mathbb{N}} \in \mathbb{K}^\mathbb{N} | \lim_{j\to\infty} x_j$ exists in $\mathbb{K}\}$.

* Exercise 8: Norm of Bounded Operator

Consider a linear operator defined on a normed space $(\mathbb{K}^n, \|\cdot\|_p)$ as $T : x \mapsto Ax$ where $A = (a_{jk})_{j,k=1}^n$ is a matrix. Calculate norm of T for

(1) $p = 1$,

(2) $p = \infty$.

* Exercise 9: Integral Operator

Consider a linear operator defined on a normed space $(C([0, 1]), \|\cdot\|_\infty)$ as $T : f \mapsto Tf(t) := \int_0^t f(\tau)d\tau$. Show the following properties

(1) $\|Tf\|_\infty \leq \|f\|_\infty$,

(2) $\|T\|_\infty = 1$,

(3) $T^n f(t) = \int_0^t \frac{(t-\tau)^{n-1}}{(n-1)!} f(\tau)d\tau$,

(4) $\|T^n f\|_\infty \leq \frac{\|f\|_\infty}{n!}$.

(5) $\|T^n\|_\infty = \frac{1}{n!}$.

Remark: The exercises marked with a star can be handed in for the correction till Wednesday noon to the letterbox at the ground level of the Math building 20.30.