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Functional Analysis

8th Exercise Sheet

* Exercise 28:

Let (X, d) be a complete metric space and $\mathcal{F} \subseteq C(X)$ such that $\{f(x) | f \in \mathcal{F}\}$ is bounded for any $x \in X$. Then there exists a ball $B(x_0, r) \subseteq X$ such that $\sup_{f\in\mathcal{F}} \sup_{x\in B(x_0,r)} |f(x)| < \infty$.

* Exercise 29:

Let X, Y be Banach spaces and let $T \in \mathcal{L}(X, Y)$. Then the range of $T(X)$ is closed in Y if and only if

$$\gamma(T) := \inf \left\{ \frac{\|Tx\|_Y}{d(x, N(T))} \Big| x \notin N(T) \right\} > 0\,.$$

Hint: Use Open Mapping Theorem.

Exercise 30:

Let $f : [1, \infty) \to \mathbb{R}$ be a continuous function such that $\lim_{n\to\infty} f(nt) = 0$ holds for all $t \geq 1$. Then $\lim_{t\to\infty} f(t) = 0$ also holds.

* Exercise 31:

Let s_n be a real sequence. Then the following are equivalent:

1. $\sum_{j=1}^{\infty} s_j$ converges absolutely,
2. series $\sum_{j=1}^{\infty} t_j s_j$ converges for every nullsequence t_n.

Hint: Use Banach–Steinhaus Theorem. There is also alternative proof using elementary results from Analysis 1.

Remark: The exercises marked with a star can be handed in for the correction till Wednesday noon to the letterbox at the ground level of the Math building 20.30.