Institut für Analysis
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Functional Analysis

9th Exercise Sheet

*** Exercise 32:**
Let X, Y, Z be Banach spaces where Y, Z are linear subspaces of a vector space W. Assume that d is a metric on W such that identities

$$I_Y : (Y, \|\cdot\|_Y) \to (W, d), y \mapsto y \quad \text{and} \quad I_Z : (Z, \|\cdot\|_Z) \to (W, d), z \mapsto z$$

are continuous. Let $T \in \mathscr{L}(X, Y)$ and suppose $T(X) \subseteq Z$. Show that $T \in \mathscr{L}(X, Z)$.

Exercise 33:
Let X, Y be Banach spaces and let $T : D(T) \to Y$ be a linear operator with $D(T) \subseteq X$. Show the following

1. T is closed $\Leftrightarrow$ $D(T)$ is a Banach space for the graph norm $\|x\|_T := \|x\|_X + \|Tx\|_Y$ for $x \in D(T)$.
2. $T \in \mathscr{L}((D(T), \|\cdot\|_T), Y)$.

*** Exercise 34:**
Let X be Banach space and $U \subseteq X$ be a closed linear subspace. Find canonical isomorphisms which shows that

$$(X/U)' \simeq U^{\perp} \quad \text{and} \quad U' \simeq X'/U^{\perp}.$$

*** Exercise 35:**
Show the following

1. Every $\mathbb{K}$-vector space has a basis.
2. If X is a normed space and $\dim X = \infty$ then there exist $\varphi : X \to \mathbb{K}$ which are linear and unbounded.

Remark: The exercises marked with a star can be handed in for the correction till Wednesday noon to the letterbox at the ground level of the Math building 20.30.