Institut für Analysis
PD Dr. Peer Christian Kunstmann
Dr. Michal Jex

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Functional Analysis

14th Exercise Sheet

*** Exercise 52:**
Let X, Y be Banach spaces and $T \in \mathcal{L}(X, Y)$. Then

$$T \text{ compact} \Rightarrow T \text{ maps weakly convergent sequences to strongly convergent sequences.}$$

Show that if X is reflexive then also reverse implication ($\Leftarrow$) holds.

*** Exercise 53:**
Let X be a reflexive space and $(x_n)_{n\in\mathbb{N}}$ be a bounded sequence in X. Show that $(x_n)_{n\in\mathbb{N}}$ has a weakly convergent subsequence.

Exercise 54:

1. Let $T : l^1 \to c_0$ be defined by $x \mapsto \left(\sum_{j=1}^{\infty} x_j, \sum_{j=2}^{\infty} x_j, \sum_{j=3}^{\infty} x_j, \ldots\right)$. Find the dual operator $T' : l^1 \to l^\infty$.
2. Let X, Y be Banach spaces and $T \in \mathcal{L}(X, Y)$ be isometric. Show that T' is surjective.
3. Let $\Omega \subseteq \mathbb{R}^n$ be bounded. Show that $\text{div} : L^2(\Omega)^n \to H^{-1}(\Omega) := (H_0^1(\Omega))^*$ is surjective.

*** Exercise 55:**
Find a bounded sequence $(x^{(n)})_{n\in\mathbb{N}}$ in c_0 such that $(x^{(n)})_{n\in\mathbb{N}}$ has no weakly converging subsequence.
Hint: Any bounded sequence in l^∞ has a weak$\star$-converging subsequence with weak$\star$-limit in $x^{(0)} \in l^\infty$. Search for $x^{(0)} \in l^\infty \setminus c_0$.

Remark: The exercises marked with a star can be handed in for the correction till Wednesday noon to the letterbox at the ground level of the Math building 20.30.