Exercise 41: Two compressed air tanks with different volumes $V_1$ and $V_2$ are linked by a pipe which is closed by a valve. Before opening the blocking valve at time $t = 0$ prevails different pressures $p_1(0)$ and $p_2(0)$ in the pressure vessels. By the ideal gas law $pV = nRT$ ($n$ amount of substance, $R$ gas constant, $T$ temperature), assuming isothermal balancing, we achieve the relation $\dot{p}V = \dot{n}RT$. Using the fluid resistance $W$ of the pipe we get the formula $\dot{n} = Wp$. With the notation $a_{1,2} := \frac{RT}{WV_{1,2}}$ we have the following system for the model:

\[
\begin{pmatrix}
\dot{p}_1(t) \\
\dot{p}_2(t)
\end{pmatrix} =
\begin{pmatrix}
-a_1 & a_1 \\
-a_2 & -a_2
\end{pmatrix}
\begin{pmatrix}
p_1(t) \\
p_2(t)
\end{pmatrix}, \quad t > 0.
\]

Let $p_1(0) = 1$ bar, $p_2(0) = 9$ bar, $a_1 = 1$ bar/s and $a_2 = 3$ bar/s.

(a) Which of the pressure vessels gets a pressure of two bar and at which time does it achieve this pressure?

(b) Which pressure will be obtained when the system is completely balanced?

Exercise 42: Consider the differential equation

\[x^2y'''(x) - 3xy''(x) + 4y'(x) = x^3 \ln x, \quad x > 0.\]

(a) Determine the general solution of the corresponding homogeneous equation.

(b) Use the method of variation of parameters to find a particular solution of the given inhomogeneous differential equation.

(c) Solve the initial value problem of this inhomogeneous differential equation with $y(1) = y'(1) = 1$.

Exercise 43: Consider the differential equation

\[y''(x) - 2y'(x) + 2y(x) = e^{2x} \sin x.\]

(a) Specify the general solution of the corresponding homogeneous equation.

(b) Find a particular solution of the differential equation using the method of undetermined coefficients.

(c) Solve the initial value problem of the given inhomogeneous differential equation, where $y(0) = 3/5, y'(0) = 1$.

Exercise 44: Solve the following differential equation using the method of undetermined coefficients:

\[y'''(x) - 3y''(x) + 4y(x) = (x^2 + 4x + 2)e^{3x} + e^{2x}, \quad x \in \mathbb{R}.\]

Exercise 45: Solve the inhomogeneous linear differential equation

\[x'(t) = Ax(t) + b(t) \quad \text{for} \quad A = \begin{pmatrix} 0 & 1 \\ -2 & 3 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} e^{-t} \\ e^{3t} \end{pmatrix}.
\]

\[ \begin{align*}
\text{(a)} & \quad b = \begin{pmatrix} e^{-t} \\ e^{3t} \end{pmatrix}, \\
\text{(b)} & \quad b = \begin{pmatrix} e^t \\ 2 \end{pmatrix}.
\end{align*}\]

Due date: Please hand in your homework until Monday, June 24, 12:00 into the post box of room D402 in the ID.
Exercise T25: Determine the characteristic polynomial of the differential equation

\[ y'''(x) + y''(x) + 4y'(x) + 4y(x) = f(x), \quad x \in \mathbb{R}. \]

For each of the following functions \( f(x) = f_i(x), i = 1, \ldots, 6 \), specify the type of a solution of the differential equation using the method of undetermined coefficients:

\[
\begin{align*}
  f_1(x) &= (4 + 12x)e^{2x} \\
  f_2(x) &= e^{-x} \\
  f_3(x) &= \sin(2x) \\
  f_4(x) &= x^2 \cos(2x) \\
  f_5(x) &= xe^{-x} \sin(2x) \\
  f_6(x) &= (2 + x) \sin(2x)
\end{align*}
\]

Use your results to find a particular solution for the cases \( f(x) = f_1(x) \) and \( f(x) = f_2(x) \).

Exercise T26: Determine the general solution of the differential equations

(a) \( y''(x) - y(x) = x, \quad x > 0 \), using the method of undetermined coefficients,

(b) \( y''(x) - y(x) = \frac{1}{2}, \quad x > 0 \), using the method of variation of parameters.

*Hint:* You don’t have to evaluate the integral \( \int x^2 dx \).

Exercise T27: Consider the differential equation

\[ x^2 y'''(x) - 3xy'(x) + y(x) = x^3, \quad x > 0. \]

(a) Show that \( y_1(x) = x^2 \) is a solution of the corresponding homogeneous equation.

(b) Find the general solution of the homogeneous equation using reduction of the order.

(c) Use the method of variation of parameters to find a particular solution of the differential equation.

(d) Determine the solution of the corresponding initial value problem with \( y(1) = \frac{17}{5} \) and \( y'(1) = \frac{21}{5} \).

For detailed information regarding this course please check the page

http://www.math.kit.edu/iana2/edu/am22013s/

Tutorial date: Thursday, June 20