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Bifurcation Theory

Problem Sheet 12

Problem 31 (Gradient systems cannot have periodic solutions)

A gradient system is a dynamical system of the form

$$x' = -\nabla_x V(x, \lambda)$$

for a given function $V \in C^1(\mathbb{R}^n \times \mathbb{R}, \mathbb{R})$.

(a) Prove that gradient systems cannot have non-constant periodic solutions.

(b) Let $V \in C^2(\mathbb{R}^n \times \mathbb{R}, \mathbb{R})$. Show that Hopf bifurcation cannot occur in gradient systems.

(c) Consider the dynamical system

$$\begin{cases} x_1' = 2 - x_1 - \lambda x_2^2, \\ x_2' = 5 - x_2 - 2\lambda x_1 x_2. \end{cases}$$

Are there periodic solutions?

Problem 32 (Langford-System)

Determine the Hopf bifurcation points $(0, \lambda_0) \in \mathbb{R}^3 \times \mathbb{R}$ of the nonlinear system

$$\begin{cases} x_1' = (\lambda - 1)x_1 - x_2 + x_1 x_3, \\ x_2' = x_1 + (\lambda - 1)x_2 + x_2 x_3, \\ x_3' = \lambda x_3 - (x_1^2 + x_2^2 + x_3^2). \end{cases}$$