4th Problem Sheet

Variational Methods and Applications to PDEs

Problem 7  (Continuation of Problem 6)
Let $\Omega$ be a bounded, open subset of $\mathbb{R}^n$ and let $f : \Omega \times \mathbb{R} \to \mathbb{R}$ and $F : \Omega \times \mathbb{R} \to \mathbb{R}$ be as in Problem 6. Prove that the functional $L : L^p(\Omega) \to \mathbb{R}$ given by

$$L[u] = \int_{\Omega} F(x, u(x)) \, dx, \quad u \in L^p(\Omega).$$

is continuously Fréchet differentiable.

Problem 8
Let $\Omega$ be a bounded and open subset of $\mathbb{R}^n$ and $a \in L^2(\Omega)$. Prove that the functional $L : H^1_0(\Omega) \to \mathbb{R}$ given by

$$L[u] = \int_{\Omega} \left( \frac{1}{2} |\nabla u|^2 + a(x) u \right) \, dx, \quad u \in H^1_0(\Omega)$$

has a unique minimizer on $H^1_0(\Omega)$.

To be discussed in the problem session on Tuesday, November 24, 2009.