Problem 22
Prove that the function \( f : \mathbb{R}^2 \rightarrow \mathbb{R} \) defined by
\[
f(x, y) = 9(x^2 + y^2) - (x^2 + y^2)^2 + y \sin(x) \quad \text{for all } (x, y) \in \mathbb{R}^2
\]
has a strictly positive critical value.

Problem 23
Let \( \Omega \neq \emptyset \) be an open and bounded subset of \( \mathbb{R}^n \) and \( 2 < p < q < 2^* \) where \( 2^* \) is defined as in Problem 11. Moreover, let the functional \( L : H^1_0(\Omega) \rightarrow \mathbb{R} \) be defined by
\[
L[u] = \int_{\Omega} \left( \frac{|\nabla u|^2}{2} - \frac{|u|^p}{p} - \frac{|u|^q}{q} \right) dx \quad (u \in H^1_0(\Omega)).
\]

a) Use the Mountain Pass Lemma to prove that \( L \) has at least one strictly positive critical value \( c \in \mathbb{R} \).

b) Let \( u^* \in H^1_0(\Omega) \) be a critical point of \( L \) such that \( L[u^*] = c \). Find a boundary value problem that is solved by \( u^* \).