A numerical verification method for the existence and uniqueness of solutions of non-linear second order Sturmian boundary value problems

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Abstract
A numerical method for verifying the existence and uniqueness of solutions of second order non-linear two-point boundary value-problems with Sturmian boundary conditions

\[-u''(x) = f(x, u(x), u'(x)), \quad \alpha < x < \beta,\]
\[\alpha_1 u(\alpha) + \alpha_2 u'(\alpha) = 0 = \beta_1 u(\beta) + \beta_2 u'(\beta)\]

in the vicinity of a given approximate numerical solution is proposed, where \(\alpha, \beta, \alpha_1, \alpha_2, \beta_1, \beta_2\) are real numbers satisfying \(\alpha < \beta\) and \((\alpha_1, \alpha_2) \neq (0,0) \neq (\beta_1, \beta_2)\). The function \(f(x, y, z)\) is assumed to be continuous on \([\alpha, \beta] \times \mathbb{R}^2\) and to be two times continuously differentiable with respect to \(y\) and \(z\). All numerical computations are carried out with INTLAB.