**Institute for Applied and Numerical Mathematics**
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# Geometric Numerical Integration

Summer Semester 2016

# Assignment 1

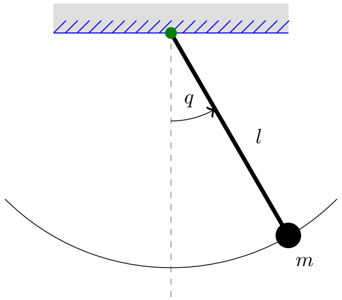
## The pendulum

Consider the *mathematical* pendulum with

- a *massless* rod of length $l = 1$
- a ball with mass $m = 1$
- gravitational acceleration $g = 1$,

and Hamiltonian

$$H(p,q) = \frac{1}{2}p^2 - \cos(q).$$




In this exercise we want to simulate the Hamiltonian equations of motion

$$\dot{q} = \partial_p H(p,y) \quad \text{and} \quad \dot{p} = -\partial_q H(p,q)$$

using each of the following numerical schemes:

- Explicit Euler method
- Implicit Euler method
- Symplectic Euler method
- Midpoint rule
- Trapezoidal rule
- Störmer-Verlet method
- Runge's method
- BDF2

(i) Write a MATLAB script that computes the approximations $q_n \approx q(t_n)$ and $p_n \approx p(t_n)$, $t_n = nh$, using each of the above numerical schemes for the time interval $[0, 4000]$, the step-size $h = 0.02$ and the initial condition $\big(q(0), p(0)\big) = (1, 0)$.

(ii) Plot for each scheme the approximation of the value $q(t)$ of the pendulum over the time interval $[3950, 4000]$ together with a reference solution generated with the built-in MATLAB function `ode45` in the same figure.

(iii) Compute the energy $H(p_n, q_n)$ for all time points $t_n = nh$, and plot the values $H(p_n, q_n) - H(p_0, q_0)$ versus time in an additional figure.

**Remarks:**

- Write one MATLAB function for each numerical method. You can (and should) implement and test the methods one at a time.
- Because some of the numerical methods to be implemented are *implicit*, a non-linear equation has to be solved in each time-step. This can be achieved using Newton iteration.

**Note:** This is a programming assignment, so there are several different correct implementations of the problem. Experiment with initial conditions or step-sizes and draw your own conclusions. You should keep the following in mind:

- Structure your code in such a way that it is easy to read.
- Use meaningful abbreviations.
- Add comments where appropriate.

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Assistance with this programming exercise will be provided in the problem sessions on **27th April, 2016** and **11th May, 2016**.

**Homepage:**

The link http://www.math.kit.edu/ianm3/lehre/geomnumint2016s/en leads to the web page of the lecture. Here you will find all up-to-date information about the lecture and the programming exercise.