Splitting Methods, Exercise sheet 1

Exercise 1:
Compute the adjoint method of

a) the explicit Euler method.
b) the implicit midpoint rule.
c) the Lie-Splitting.
d) the Strang-Splitting.

Exercise 2:
Prove that the Lie splitting method applied to the initial value problem

\[ y'(t) = f(y(t)) = f^1(y(t)) + f^2(y(t)), \quad 0 < t < T, \quad y(0) = y_0 \]  \tag{1}

with \( f : \mathbb{R} \to \mathbb{R} \) and \( f^i : \mathbb{R} \to \mathbb{R}, i = 1, 2 \) is consistent of order 1. In particular state the regularity assumptions on \( f \) and \( f^i, i = 1, 2 \). Furthermore precisely state the error structure (i.e., under which conditions on \( f^i, i = 1, 2 \) do we obtain consistency of order \( p > 1 \)?)

Exercise 3:
Prove that the Strang splitting method applied to (1) is consistent of order 2 and conclude a lemma for this result (see Lemma 1.1). In particular state the regularity assumptions on \( f \) and \( f^i, i = 1, 2 \). Furthermore precisely state the error structure (i.e., under which conditions on \( f^i, i = 1, 2 \) do we obtain consistency of order \( p > 2 \)?)

Exercise 4:
Consider the following model problem:

\[ y'(t) = Ly(t) = Ay(t) + By(t), \quad y(0) = y_0 \in \mathbb{R}^n, \quad t \in [0, T], \]

with matrices \( L, A, B \in \mathbb{R}^{n \times n} \).

Prove that the local error of the Lie splitting method is bounded by

\[ \| y(h) - e^{hA}e^{hB}y_0 \| \leq C h^2 \| [A, B] \| . \]

Will be discussed in the exercise class on: 05.11.2013.