Splitting Methods, Exercise sheet 6

**Exercise 1:**

Interpret the leapfrog method (Störmer Verlet scheme) as a numerical time-integrator for second order odes as a splitting method.

**Reference:**


**Exercise 2:**

Consider the linear Schrödinger equation

\[ i \partial_t u = -\Delta u + V(x)u, \quad x \in \mathbb{T} = (-\pi, \pi) \]

\[ u(0) = \cos(x) \]

with periodic boundary conditions and \( V(x) = \cos(x) \).

a) Implement the Strang splitting method which is defined by

\[ u_n = \left( e^{-i \frac{\pi}{2} V(x)} e^{i \Delta \tau} e^{-i \frac{\pi}{2} V(x)} \right)^n u_0 \approx e^{-i (-\Delta + V)n \tau} u_0 = u(t_n) \]

as a numerical time-integrator. Use a pseudo-spectral method to discretize the problem in space. Compute therefore the exact solution of

\[ i \partial_t u = -\Delta u \]

in Fourier space.

b) Compute a reference solution at \( T = 1 \) and check the order of this splitting method numerically.

Will be discussed in the exercise class on: 11.02.2014.