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Wavelets - Theory and Applications

Problem Set 13

39. Let ψ be an orthogonal wavelet associated with the orthogonal scaling function φ. Let ϱ denote a 2π-periodic function satisfying $|\varrho(\omega)| = 1$, $\omega \in \mathbb{R}$. Then, $\widetilde{\psi}$ defined by

$$\widehat{\widetilde{\psi}} = \varrho\, \widehat{\psi},$$

is also an orthogonal wavelet associated with φ.

40. Let $\{h_k\}_{0 \le k \le 2N-1}$ be scaling coefficients of the Daubechies scaling function of order N. Let $g_k = (-1)^k h_{2N-1-k}$, $k = 0, \dots, 2N-1$, be the corresponding wavelet coefficients. Then,

$$\sum_{k=0}^{2N-1} k^m g_k = 0, \quad m = 0, \dots, N-1.$$

41. Let DW_N be the Daubechies wavelet of order N. Show:

$$\int_{\mathbb{R}} t^m \, \mathrm{DW}_N(t)\, \mathrm{d}t = 0, \quad m = 0, \dots, N-1.$$

42. Let DS_N be the Daubechies scaling function of order N. Then, for any $m \in \{0, \dots, N-1\}$ we have that

$$x^m = \sum_{k \in \mathbb{N}} c_{m,k} \mathrm{DS}_N(x-k) \text{ pointwise for any } x \in \mathbb{R},$$

where

$$c_{m,k} = \int_{\mathbb{R}} (t+k)^m \mathrm{DS}_N(t)\, \mathrm{d}t.$$