Problem T56: Compute the following limits using L'Hôpital's rule:

\[ a) \lim_{x \to 0} \frac{\tan x - x}{x^3} \]
\[ b) \lim_{x \to \infty} \left( x + \frac{1}{\ln(1 - \frac{1}{x})} \right) \]
\[ c) \lim_{x \to \frac{\pi}{2}} \frac{\ln\left(\frac{\pi}{2} - x\right)}{\tan x} \]
\[ d) \lim_{x \to 0} x^{\tan x} . \]

Problem T57: Given the function \( f : \mathbb{R} \to \mathbb{R}, \)
\[ f(x) = \begin{cases} 
 1 - x + \ln x, & x \neq 1 \\
 1, & x = 1 
\end{cases} \]
determine \( c \) such that \( f \) is continuous at the point \( x = 1. \)

Problem T58: Using the mean value theorem, prove the following inequalities:

a) \( |\cos e^x - \cos e^y| \leq |x - y| \) für \( x, y \leq 0 \)
b) \( \ln(1 + x) \leq \frac{x}{\sqrt{1 + x}} \) für \( x > 0. \)

Hint b): Consider \( f(t) = \ln(1 + t) - \frac{t}{\sqrt{1 + t}} \) in the interval \([0, x]).\)

Problem T59: Let \( f(x) = x^x, x > 0. \) Why is \( f \) differentiable? Determine \( f' \), the monotonicity and the extreme values of \( f. \)

Problem T60: a) Give all the points at which the function \( f(x) = x + 2\sin x, x \in \mathbb{R}, \)
satisfies the condition \( f'(x) = 0. \) Which of these are actually extreme points?
b) Carry out a similar analysis for the function \( g(x) = x + \sin x, x \in \mathbb{R}. \)

Tutorial date: Tuesday, February 7, 2006, 8:00 am