Tutorial 14
Advanced Mathematics I for Mechanical Engineering

Problem T61: Prove by induction that for every \( n \in \mathbb{N} \),
\[
\sum_{k=1}^{n} \frac{1}{\sqrt{k}} \geq \sqrt{n}.
\]

Problem T62: Define the function \( f \) by
\[
f(x) = \log x^2 + 1 + x, \quad x \in \mathbb{R}.
\]
Using the Mean Value Theorem, show that \( f \) is Lipschitz continuous with Lipschitz constant \( L = 2 \).

Problem T63: Let \( \lambda \in \mathbb{R} \) be a parameter and
\[
a_n = (\lambda + n)^2 \cosh n e^{\lambda n}, \quad n = 0, 1, 2, 3, ...
\]
Determine all values of \( \lambda \in \mathbb{R} \), for which the power series
\[
\sum_{k=0}^{\infty} a_n x^n
\]
has radius of convergence \( e \).

Problem T64: Let the plane \( E \subset \mathbb{R}^3 \) and the points \( u \in \mathbb{R}^3 \) and \( v \in \mathbb{R}^3 \) be defined as follows
\[
E = \{ x \in \mathbb{R}^3 : x_1 - 2x_2 + 2x_3 = 4 \} \subset \mathbb{R}^3,
\]
\[
u = (2, 3, 1)^\top \in \mathbb{R}^3,
\]
\[
u = \frac{1}{9}(10, -2, -7)^\top \in \mathbb{R}^3.
\]
a) Determine the line through \( u \) that is orthogonal to \( E \).
b) What is the distance between \( u \) and \( E \)?
c) Determine the vectors \( a^{(1)}, a^{(2)} \in \mathbb{R}^3 \) of length 3, whose orthogonal projection onto the plane \( E \) is \( v \).

Problem T65: Solve the equation
\[
\sqrt{2} \sinh z + \cosh z = i
\]

Tutorial date: Wednesday, February 16, 2006, 14:00 pm