Worksheet 10
Advanced Mathematics I for Mechanical Engineering

Problem 46: Determine the radius of convergence of the following power series:

(a) \( \sum_{k=0}^{\infty} \frac{k + 2}{2^k} x^k \),  
(b) \( \sum_{k=0}^{\infty} \frac{x^{2k}}{(2 + \frac{1}{k})^k} \),  
(c) \( \sum_{k=0}^{\infty} \frac{3^k + 2}{2^k} x^k \).

Problem 47: For which \( x \in \mathbb{R} \) does the following power series converge?

\( \sum_{n=1}^{\infty} \frac{1}{n^2} \left( \sqrt{n^2 + n} - \sqrt{n^2 + 1} \right)^n (x + 1)^n \)

Problem 48: Let

\( f(x) = \sum_{n=1}^{\infty} \frac{1}{n!} x^{n-1} \)

for \( x \in (-2\pi, 2\pi) \). Compute the first five coefficient \( b_n \) of the power series

\( \sum_{n=0}^{\infty} b_n x^n = \frac{1}{f(x)} \).

Problem 49: Determine all \( z \in \mathbb{C} \), which satisfy the equation

\( \cos z = 4 \)

Use the exponential representation of the cosine function.

Problem 50: Find all solutions \( z \in \mathbb{C} \) of the equation

\( \cosh z - \frac{1}{2} (1 - 8i) e^{-z} = 2 + 2i \).

Due date: Tuesday, January 24, 2006, 8:00 am (in the tutorial)
Tutorial 10
Advanced Mathematics I for Mechanical Engineering

Problem T37: Determine the radius of convergence of the power series:

(a) \([\sum_{k=0}^{\infty} \frac{z^k}{3(k+2)!}]\),
(b) \([\sum_{k=1}^{\infty} \frac{z^{2k} \cdot 2^k}{(1 + \frac{1}{k})^k}]\),
(c) \([\sum_{k=0}^{\infty} k^k z^k]\).

Problem T38: For which \(x \in \mathbb{R}\) does the following power series converge?

\[\left(\sum_{n=0}^{\infty} \frac{n+1}{\sqrt[3]{8^n}} \left(\frac{n}{2}\right) (x - 2)^{3n}\right)\]

Problem T39:
(a) Compute the power series of the rational function \(f : \mathbb{C} \setminus \{1\} \rightarrow \mathbb{C}\), where

\[f(z) = \frac{1 + z^2}{1 - z}\]

at the center of expansion \(z_0 = 0\).
(b) Compute the radius of convergence of the series.
(c) For which \(z \in \mathbb{C}\) does the power series converge?

Problem T40: For each of the following equations, determine the set of solutions \(z \in \mathbb{C}\):

(a) \(\cos z = \cos \bar{z}\)  \hspace{1cm}  (b) \(e^{zi} = e^{i\bar{z}}\)

Tutorial: Tuesday, January 17, 2006, 8:00 am, room 203 in the ID