Worksheet 4
Advanced Mathematics I for Mechanical Engineering

Problem 16: Find the values of $\alpha \in \mathbb{C}$, for which the solutions of the linear system of equations

\[
(i\alpha - i)z_1 + i\alpha^2 - i)z_2 -(1 + i)z_3 = -2 - 2i
\]
\[
(i - 1)z_1 + (i - 1)z_2 - iz_3 = -1 - i
\]
\[
z_1 + z_2 - z_3 = -1
\]

(a) don’t exist;
(b) form a straight line in $\mathbb{C}^3$. Determine this straight line.

Problem 17: Find all solutions of the linear system of equations

\[
3x_1 + x_3 - x_4 = 4
\]
\[
-8x_1 + \alpha x_2 -(\alpha + 2)x_3 + x_4 = -8
\]
\[
-8x_1 -(2\alpha + 6)x_3 + (2\alpha + 5)x_3 - 2x_4 = 2\beta
\]
\[
-4x_1 -(2\alpha + 2)x_2 + 2\alpha x_3 + 3x_4 = -1 + \beta
\]

depending on the parameters $\alpha, \beta \in \mathbb{R}$.

Problem 18: Let

\[
g_1 : x = \begin{pmatrix} 3 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}, \quad \lambda \in \mathbb{R}, \quad g_2 : y = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad \mu \in \mathbb{R}.
\]

be straight lines in $\mathbb{R}^3$.

(a) Do they intersect each other? Find either their intersection point or the distance between them.
(b) Determine the plane $E_1$, which contains $g_1$ and is orthogonal to $g_2$ and the plane $E_2$, which contains $g_2$ and is orthogonal to $g_1$. What is the angle between them?

Problem 19: Consider the plane $E : 4x_1 + x_3 + 8 = 0$, the point $P = (2, 1, 1)^T$ and the line $h : x = (4, 3, -2)^T + \lambda(3, 1, -1)^T, \lambda \in \mathbb{R}$.

(a) Determine a line through $P$ that is orthogonal to $E$.
(b) Determine the distance from $P$ to $E$ as well as the point $Q$ in $E$ closest to $P$.
(c) Determine the point at which the line $h$ intersects $E$ and the point $R$ on $h$, that is closest to $P$.

Problem 20: Let $x^{(1)}, x^{(2)}, x^{(3)} \in \mathbb{R}^3$. Show that the following statements are equivalent, i.e. each one of them follows from the other one:

- The vectors $x^{(1)}, x^{(2)}$ and $x^{(3)}$ have a length of 1 and are pairwise orthogonal.
- Each vector $x \in \mathbb{R}^3$ can be represented as $x = (x \cdot x^{(1)}) x^{(1)} + (x \cdot x^{(2)}) x^{(2)} + (x \cdot x^{(3)}) x^{(3)}$.

Due date: Tuesday, November 29, 8:00 (in the tutorial)
Tutorial 4
Advanced Mathematics I for Mechanical Engineering

**Problem T1:**
(a) For which values of $\alpha$ and $\beta \in \mathbb{R}$ is the following linear system $Ax = b$,

$$
A = \begin{pmatrix}
2 & 3 & 0 & 1 \\
1 & 1 & 0 & -1 \\
0 & 2 & 1 & 3 \\
3 & -1 & 2 & \alpha \\
\end{pmatrix}, \quad b = \begin{pmatrix}
\beta \\
1 \\
0 \\
2 \\
\end{pmatrix}
$$

solvable?

(b) Give the set of solutions $L$ of $Ax = b$ and the set of solutions $L_0$ of the homogeneous system $Ax = 0$.

**Problem T2:** Let

$$
x = \begin{pmatrix}
1 \\
2 \\
\end{pmatrix} \quad \text{and} \quad y = \begin{pmatrix}
\alpha \\
2 \\
\end{pmatrix}
$$

be vectors in $\mathbb{R}^2$ and $\alpha \in \mathbb{R}$ is a parameter. We are looking for a matrix $A \in \mathbb{R}^{2 \times 2}$, such that $Ax = y$ and $Ay = x$. For which $\alpha$ can such $A$ be uniquely determined? Compute the matrix for this case.

**Problem T3:** Compute the distance between the two skew straight lines $g_1 : x = (1, 2, 3)^\top + \lambda(1, -1, 1)^\top$ and $g_2 : x = (2, 2, 1)^\top + \mu(0, 1, 1)^\top$ and the corresponding perpendicular feet $F_1$ and $F_2$.

**Problem T4:** Given the straight line

$$
g : x = \begin{pmatrix}
1 \\
2 \\
0 \\
\end{pmatrix} + s \begin{pmatrix}
1 \\
-1 \\
2 \\
\end{pmatrix}, \quad s \in \mathbb{R},
$$

and the points $P = (2, 0, 2)^\top$ and $Q = (0, 2, 2)^\top$,

(a) determine a parameter equation of the straight line $h$, which passes through $P$ and $Q$;

(b) determine a point $R$ on $g$ such that the plane $E_1$ containing $P, Q$ and $R$ is parallel to the plane $E_2 : 2x_1 + 2x_2 - 3x_3 = 0$.

(c) determine the point $S$ on $g$, which lies at the same distance to $E_1$ and $E_2$.

**Tutorial date:** Tuesday, November 22, 2005, 8:00 am, room 203 in the ID