Worksheet 8
Advanced Mathematics I for Mechanical Engineering

**Problem 36:** Show that the polynomial
\[ p(x) = x^5 - 9x^4 - \frac{82}{9}x^3 + 82x^2 + x - 9 \]
has exactly 3 zeroes in the interval \([-1, 4]\).

**Problem 37:** Given the functions \( f, g : \mathbb{R} \to \mathbb{R} \),
\[
 f(x) = \begin{cases} 
 4 - x^2, & x \leq 2, \\
 4x^2 - 24x + 36, & x > 2, 
\end{cases}
\]
and \( g(x) = x + 1 \),
show that their graphs have at least 4 intersection points.

**Problem 38:** Are the following subsets of \( \mathbb{C} \) open, closed, bounded, or compact? Justify your answer.
(a) \( A = \{ z \in \mathbb{C} : 0 < \text{Re}(2 + z + \bar{z} + z\bar{z}) \leq 2 \} \)
(b) \( B = \{ z \in \mathbb{C} : 0 \leq \text{Im}\left( (1 + i)\bar{z} + \frac{1 + z^2\bar{z}^2}{(z - \bar{z})^2 - 1} \right) \leq 1 \} \)

**Problem 39:** The set \( M \) is defined as follows:
\[ M := \{ x \in \mathbb{R} : \text{there is an } n \in \mathbb{N}, \text{s.th. } x \in I_n \}, \]
where \( I_n = \left[ \frac{1}{n+1}, \frac{1}{n} \right] \), \( n \in \mathbb{N} \). Show that \( M \) is not compact and find a continuous function over \( M \),
which does not attain its maximum in \( M \).

**Problem 40:** Let the set of complex numbers \( D \) be defined as \( D := \{ z \in \mathbb{C} : |z| \leq 1 \text{ und } \text{Im}(z) \geq 0 \} \).
Consider the function \( f : D \to \mathbb{R} \),
\[ f(z) = |\text{Im}((2 - i)z)|. \]
Are there points in \( D \), where \( f \) assumes its maximum and minimum? If so, find them and compute the corresponding function values.

**Due date:** Tuesday, February 1, 2006, 8:00 am, (in the tutorial).
Problem T29: Which of the following sets are bounded, open, closed, or compact?
(a) $M_1 = [-1, 42]$
(b) $M_2 = (-1, 42]$
(c) $M_3 = (-1, 42)$
(d) $M_4 = (-\infty, \infty)$
(e) $M_5 = \{ z \in \mathbb{C} : -1 \leq \text{Im } z \leq 1 \}.$

Problem T30: Are the following subsets of $\mathbb{C}$ open, closed, bounded, or compact? Justify your answer.
(a) $A = \{ z \in \mathbb{C} : 0 < \text{Re} \left( \frac{z^2 + 2}{z^2 + 1} \right) < 3 \}$
(b) $B = \{ z \in \mathbb{C} : \text{Re}(z^2 \bar{z}^2 + z - 5z\bar{z} - \bar{z} + 4) < 0 \}$

Problem T31: Find the number of solutions of the equation
$$2x^5 - 6x^3 + 2x = 4x^4 - 6x^2 + 1$$
in the interval $I = [-2, 2]$ and justify your answer.

Problem T32: Show that at any given time there are two antipodal points on the equator at which the temperature is exactly the same.

Hint: Suppose that the temperature on the equator is a continuous function $\tau : [0, 2\pi] \to \mathbb{R}.$ Construct a function that represents the temperature difference between antipodal points, and see if you can apply the Intermediate Value Theorem.

Tutorien: Tuesday, December 20, 2005