Worksheet No.1
Advanced Mathematics I

Exercise 1: Let $A, B \subseteq \mathbb{R}$ be defined as

$A := \{ x \in \mathbb{R} : |x^2 - 2| \leq 4 - x \}$ and $B := \{ x \in \mathbb{R} : 1 - |x - 2| < \frac{1}{2} |x - 3| \}$.

Give a representation of $A \cup B$, $A \cap B$ and $A \setminus B$ as intervals.

Exercise 2: Evaluate the following sums:

(a) $\sum_{n=7}^{42} \left( \frac{1}{3} \right)^n$, (b) $\sum_{m=1}^{10} (n + 1)^3$, (c) $\sum_{\mu=0}^{4} \sum_{\nu=2}^{4} \frac{1}{\mu + \nu^2}$.

Exercise 3: Use mathematical induction to show

(a) $2^n \geq n^2$, $n \in \mathbb{N}_0$, (b) $\sum_{k=1}^{n} \frac{1}{(2k - 1)(2k + 1)} = \frac{n}{2n + 1}$, $n \geq 1$.

Exercise 4: Use mathematical induction to show

(a) $\sum_{k=1}^{n} \frac{1}{n + k} \geq \frac{1}{2}$, (b) $\sum_{\ell=0}^{n} \binom{n}{\ell} = 2^n$.

Exercise 5: A ball falls to the ground from an initial height $h_1$ and then bounces back to a height $h_2 = \frac{3}{4} h_1$, falls back down and then bounces to a height $h_3 = \frac{3}{4} h_2$, and so on. What is the total distance the ball will travel till the 10th contact with the ground?

Due date: Please hand in your homework on Friday, November 9, 11:15.
Exercise T1: Consider the sets

\[ I := \left\{ x \in \mathbb{R} : \frac{1-x}{3-x} \leq 0 \right\} \quad \text{and} \quad J := \{ x \in \mathbb{R} : |x+1| + |x-1| > 3 \}. \]

(a) Give a representation of I and J as intervals.
(b) Determine \( I \cap J \) and \( J \setminus I \).

Exercise T2: Calculate the following sums:

\( \sum_{n=1}^{63} n \), \( \sum_{n=1}^{8} (n - 1/2)^2 \), \( \sum_{\nu=1}^{4} \sum_{k=1}^{\nu} \nu (\nu - k) \).

Exercise T3: Use mathematical induction to show

(a) \( n^2 \geq 2n + 1, \quad n \geq 3, \quad n \in \mathbb{N} \),
(b) \( \sum_{k=0}^{n} (2k + 1) = (n + 1)^2, \quad n \in \mathbb{Z}_{\geq 0} \).

Exercise T4: Use mathematical induction to show:

(a) \( \sum_{j=1}^{n} (-1)^{j-1} j^2 = (-1)^{n-1} \binom{n+1}{2} \), \( n \in \mathbb{N} \),
(b) \( \sum_{k=1}^{2n} \frac{1}{k} \geq 1 + \frac{n}{2}, \quad n \in \mathbb{N} \).

Tutorial date: Tuesday, November 6, 2007.