Worksheet No. 3
Advanced Mathematics I

Exercise 11:
(a) Show that the lines
\[ G : x(t) = (1, 2, 5)^\top + t(-1, 1, 2)^\top, \quad t \in \mathbb{R} \] and
\[ H : y(s) = (3, -1, 2)^\top + s(1, 1, 0)^\top, \quad s \in \mathbb{R} \]
are skew, i.e. are not parallel and have no common point.

(b) Determine the equation of the plane that is parallel to \( G \) and \( H \) and is the same distance from each of them.

Exercise 12: Let \( E := \{x \in \mathbb{R}^3 : x_1 - x_3 = 0\} \) and \( F \) : a plane passing through the points \( A = (4|0|0), \) \( B = (3|0|1), \) \( C = (2|1|0) \) two subsets of \( \mathbb{R}^3. \)

(a) Find the parametric form of the sets \( E \) and \( F. \)

(b) Determine the intersection \( G = E \cap F. \)

(c) Which of the sets \( G, E, F \) is a subspace of \( \mathbb{R}^3? \) Give reason for your answer.

Exercise 13: Solve the following non homogeneous linear systems of equations by using Gaussian Elimination:

(a) \[
\begin{align*}
x + 2y - 3z &= -1 \\
3x - y + 2z &= 7 \\
5x + 3y - 4z &= 2
\end{align*}
\]
(b) \[
\begin{align*}
2x + y - 2z &= 10 \\
3x + 2y + 2z &= 1 \\
4x + 3y - 2z &= 14
\end{align*}
\]
(c) \[
\begin{align*}
x + 2y - 3z &= 6 \\
2x + y + 5z &= -6 \\
2x + y + 2z &= 0
\end{align*}
\]

Exercise 14: Determine all solutions of the linear systems

(a) \[
\begin{align*}
x_1 + 3x_2 &= x_3 + x_4 = 1 \\
2x_1 + x_2 + 16x_3 + x_4 &= 11 \\
-x_1 + 2x_2 + 2x_4 &= 0 \\
x_2 + 3x_3 + x_4 &= 2
\end{align*}
\]

(b) \[
\begin{align*}
(1 + i)x_1 - ix_2 &= 2 + 3i \\
(2 + i)x_1 + (3 - i)x_2 &= 4 + 7i
\end{align*}
\]

Exercise 15: Determine all solutions of the following linear systems of equations:

\[
\begin{align*}
\alpha^2 x + (2\alpha^2 - 4)y + (2\alpha^2 + 1)z &= \alpha - 10 \\
2x + y + 5z &= -6 \\
\alpha^2 x + (2\alpha^2 + 1)y + 2\alpha^2 z &= \alpha + 2
\end{align*}
\]

Due date: Please hand in your homework on Friday, November 23, 11:15.
Exercise T9:
(a) Do the following lines intersect:
\[ G_1 : x(\lambda) = (-2, 5, 1)^\top + \lambda(3, -4, 2)^\top, \lambda \in \mathbb{R} \]
and
\[ G_2, \text{ which passes through the points with position vectors } (1, 3, -4)^\top, (0, 5, -7)^\top \text{ and } (2, 1, -1)^\top? \]

(b) Determine the line of intersection of the planes
\[ E_1 : x(\alpha, \beta) = (2, 0, 0)^\top + \alpha(1, 2, 0)^\top + \beta(0, 4, 1)^\top, \alpha, \beta \in \mathbb{R}, \]
\[ E_2, \text{ which contains the points with position vectors } (1, 0, 0)^\top, (3, 0, 1)^\top \text{ and } (1, 2, 3)^\top. \]

Exercise T10: Solve the following non homogeneous linear systems of equations by using Gaussian Elimination:

(a) \[
\begin{align*}
2x + y - 2z &= 10 \\
3x + 2y + 2z &= 1 \\
5x + 4y + 3z &= 4
\end{align*}
\]

(b) \[
\begin{align*}
(1 + 3i)z_1 + (2 + 3i)z_2 &= -12 + 11i \\
(-1 + 2i)z_1 + (1 + 2i)z_2 &= -9 + 6i
\end{align*}
\]

Exercise T11: Determine all solutions of the following linear systems of equations:

(a) \[
\begin{align*}
-3x_1 + x_2 + x_3 &= 3 \\
-2x_1 - 2x_2 + x_3 &= 1 \\
-2x_1 - x_2 + x_3 &= 2
\end{align*}
\]

(b) \[
\begin{align*}
x_1 + 2x_2 + 4x_3 &= 3 \\
4x_1 + 7x_2 + x_3 &= 2 \\
-2x_1 - 3x_2 + 7x_3 &= 4
\end{align*}
\]

(c) \[
\begin{align*}
-5x_1 + 6x_2 + 5x_3 &= 1 \\
5x_1 - 9x_2 - 5x_3 &= 0 \\
2x_1 + 5x_2 - 2x_3 &= 2
\end{align*}
\]

(d) \[
\begin{align*}
4x_1 + 4x_2 - 5x_3 &= -1 \\
4x_1 - 3x_2 - 9x_3 &= 2 \\
-3x_1 - 6x_2 + 2x_3 &= 2 \\
-6x_1 - 7x_2 + 7x_3 &= 2
\end{align*}
\]

Exercise T12: Let \[ G : x(s) = (5, 1, -1)^\top + s(4, 0, -3)^\top, s \in \mathbb{R} \text{ a line and } P_\alpha = (0|2|4\alpha), \alpha \in \mathbb{R} \text{ und } Q = (0|2|2). \]

(a) Determine the parameter form of the line \( H_\alpha \) passing through the points \( P_\alpha \) and \( Q \).

(b) Are \( G \) and \( H_\alpha \) subspaces of \( \mathbb{R}^3 \)? Give reason for your answer.