Tutorial 13
Advanced Mathematics I

Exercise T46: Solve the complex equations

(a) \((\sinh(iz) + \cosh(iz))^2 + 2 \sin(2z) = 0\),
(b) \(\sinh(iz) + \cosh(iz)) \sin(2z) = \sqrt{2}(i \sin(z) + \cos(z))\).

Solve the quadratic equation by completing the square.

Exercise T47: The sequence \((a_n)\) is defined recursively by the initial value \(a_0\) and the iteration formula

\[ a_{n+1} = \frac{1}{2 - a_n}, \quad n > 0. \]

(a) Show that the sequence is well defined and convergent for \(a_0 < 1\) and compute its limit.
(b) What is the difference, when \(a_0 = 1\), \(a_0 = \frac{5}{4}\), and \(a_0 = \frac{100\pi}{99}\) ?

Exercise T48:

(a) Compute the power series of the rational function \(f : C \setminus \{1\} \to C\), where \(f(z) = \frac{1 + z^2}{1 - z}\) at the center of expansion \(z_0 = 0\).
(b) Compute the radius of convergence of the series.
(c) For which \(z \in C\) does the power series converge?

Exercise T49: Determine the indefinite integral

\[ \int \frac{2}{\tan(\frac{x}{2}) + \cos(x) - \sin(x)} dx \]

using the substitution \(u = \tan(\frac{x}{2})\).

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