Worksheet No.1
Advanced Mathematics I

Exercise 1: Let $A, B \subseteq \mathbb{R}$ be defined as
\[ A := \{ x \in \mathbb{R} : |x^2 - 2| \leq 4 - x \} \quad \text{and} \quad B := \{ x \in \mathbb{R} : 1 - |x - 2| < \frac{1}{2} |x - 3| \}. \]
Give a representation of $A \cup B$, $A \cap B$ and $A \setminus B$ as intervals.

Exercise 2: Evaluate the following sums:
\begin{align*}
 (a) \sum_{n=1}^{42} \left( \frac{1}{3} \right)^n, \quad & (b) \sum_{m=-1}^{8} (n + 1)^3, \quad & (c) \sum_{\mu=0}^{1} \sum_{\nu=2}^{4} \frac{1}{\mu + \nu^2}.
\end{align*}

Exercise 3:
(a) Evaluate the following sums:
\begin{align*}
 (i) \sum_{k=0}^{5} \left( \begin{array}{c} 5 \\ k \end{array} \right), \quad & (ii) \sum_{n=3}^{5} \left( \begin{array}{c} n \\ 3 \end{array} \right).
\end{align*}
(b) Prove that for $n \geq m \geq r \geq 0$ it holds
\[ \left( \begin{array}{c} n \\ m \end{array} \right) \cdot \left( \begin{array}{c} m \\ r \end{array} \right) = \left( \begin{array}{c} n \\ r \end{array} \right) \cdot \left( \begin{array}{c} n - r \\ m - r \end{array} \right). \]

Exercise 4: Determine all solutions of the following system of linear equations:
\begin{align*}
 (a) \begin{cases} -6x_1 - 9x_2 + x_3 = -8 \\ -6x_1 - 7x_2 - x_3 = -4 \end{cases} \quad & (b) \begin{cases} 5x_1 + 3x_2 - 2x_3 = 2 \\ -2x_1 - 2x_2 + 3x_3 = 0 \\ -8x_1 - 2x_2 - 5x_3 = -4 \end{cases} \\
 (c) \begin{cases} -3x_1 + 4x_2 - 3x_3 = -5 \\ 3x_1 - 2x_2 + 3x_3 = 7 \\ -2x_1 + 4x_2 - 2x_3 = -1 \end{cases}
\end{align*}

Exercise 5: Find all solutions of the linear system of equations
\begin{align*}
3x_1 + x_3 - x_4 &= 4 \\
-8x_1 + \alpha x_2 - (\alpha + 2)x_3 + x_4 &= -8 \\
-8x_1 - (2\alpha + 6)x_2 + (2\alpha + 5)x_3 - 2x_4 &= 2\beta \\
-4x_1 - (2\alpha + 2)x_2 + 2\alpha x_3 + 3x_4 &= -1 + \beta
\end{align*}
depending on the parameters $\alpha, \beta \in \mathbb{R}$.

Due date: Please hand in your homework on Thursday, November 5, 12:00, into the AM1-box near Seminar room 1C-03, Allianz-Gebäude (05.20).
Exercise T1: Determine the set of all \( x \in \mathbb{R} \), that fulfill the following condition:

(a) \( \frac{(x + 1)(3 - x)}{(x + 5)^2} \leq 0 \),

(b) \( |x|^3 = x^3 + 2x^2 - 3x \).

Exercise T2: Evaluate the following sums:

(a) \( \sum_{n=17}^{63} n \),

(b) \( \sum_{n=1}^{8} (n - 1/2)^2 \),

(c) \( \sum_{\nu=1}^{4} \sum_{k=1}^{\nu} \nu (\nu - k) \).

Exercise T3:

(a) Evaluate the following sums:

(i) \( \sum_{n=1}^{5} \left( \frac{5}{4} \right) \),

(ii) \( \sum_{n=4}^{7} \left( \frac{n}{4} \right) \).

(b) Prove that for \( n \geq r \geq 1 \) it holds

\[
\binom{n}{r} = \frac{n}{r} \cdot \binom{n-1}{r-1}.
\]

Exercise T4:

(a) Determine the solution of the system of linear equations

\[
\begin{align*}
\quad & x_1 - x_2 + x_3 + x_4 = 0 \\
2x_1 & + x_2 - x_3 + 2x_4 = 0 \\
3x_1 & + 2x_2 + x_3 = 3
\end{align*}
\]

(b) For which real numbers \( \alpha \) and \( \beta \) does the following system of linear equations have (i) a unique solution, (ii) more than one solution and (iii) no solution?

\[
\begin{align*}
\alpha x_1 + x_2 + 2x_3 & = 1 \\
-x_1 + 3x_2 + x_3 & = \beta \\
2x_1 & + 2x_3 = 2
\end{align*}
\]

For detailed information regarding this course please check the page

http://www.mathematik.uni-karlsruhe.de/iag1/lehre/am12009w/en

Tutorial date: Tuesday, October 27, 2009, 3:45-5:15 pm.