Worksheet No.6
Advanced Mathematics II

Exercise 26: Consider the following differential equation
\[ u''(x) + Bu'(x) + 4u(x) = 8 \sin(x) \cos(x), \quad B \in \mathbb{R}. \]
(a) Determine \( B \) so that \( u_p(x) = 4 \cos^2(x) - 2 \) is a solution.
(b) With the constant \( B \) from part (a) determine the real-valued general solution of the differential equation.

Exercise 27: Solve the initial value problem
\[ y'''(x) + 3y''(x) + 4y'(x) - 8y(x) = (7 - 13x)e^x \]
with \( y(0) = y''(0) = 2 \) and \( y'(0) = 0 \).

Exercise 28: For the nonhomogeneous linear differential equation of 2nd order
\[ y''(x) - 3y'(x) - 2y(x) = -18x \sin(x), \quad x \in \mathbb{R}, \]
find a particular solution by the method of undetermined coefficients.

Exercise 29: Consider the nonhomogeneous linear differential equation
\[ (1 + x^2)u''(x) - 2xu'(x) + 2u(x) = 3x + x^3, \quad x \in (0, \infty). \]
Determine the general solution of the differential equation by the method of variation of parameters.
Hint: You can use that \( \{x, x^2 - 1\} \) is a fundamental system of the homogeneous differential equation.

Exercise 30: Consider the following differential equation
\[ 4u(x) - 2xu'(x) + x^3u^{(3)}(x) = 9, \quad x > 0. \]
(a) Determine a real fundamental system.
(b) Determine a particular solution by the method of variation of parameters.
(c) Solve the initial value problem with \( u(1) = \frac{13}{4} \), \( u'(1) = \frac{27}{4} \) and \( u^{(2)}(1) = \frac{50}{4} \).

Due date: Please hand in your homework on Wednesday, May 28, 09:30.
Aufgabe T17: Determine the general solution of the differential equation

\[ y''(x) - 4y'(x) + 13y(x) - (10x - 2)e^x = 0. \]

Aufgabe T18: Determine the roots of the characteristic polynomials and an ansatz for a particular solution by the method of undetermined coefficients of the following differential equations:

(a) \( y''(x) + y(x) = x \sin x, \)
(b) \( y'''(x) - 4y''(x) - 2y'(x) + 20y(x) = x^2 e^x, \)
(c) \( y'''(x) + 6y''(x) + 12y'(x) + 8y(x) = x e^{-2x}, \)
(d) \( y'''(x) + y''(x) - 6y'(x) = x e^{2x} + 2 e^{-3x}, \)
(e) \( y^{(4)}(x) + 4y'''(x) + 6y''(x) + 4y'(x) + 5y(x) = -8 \cos x - 8 \sin x, \)
(f) \( y^{(5)}(x) + y^{(4)}(x) - 4y'''(x) - 16y''(x) - 20y'(x) - 12y(x) = e^{-3x}. \)

Hint: A solution of the homogeneous differential equation in (e) is \( y(x) = x \cos x \) and one solution of the homogeneous differential equation in (f) is \( y(x) = x \sin(x) e^{-x}. \)

Aufgabe T19: Consider the linear non-homogeneous differential equation

\[ (1 - x)y''(x) + xy'(x) - y(x) = (1 - x)^2. \]

Determine the general solution by the method of variation of parameters.

Hint: You can use that \( \{x, e^x\} \) is a fundamental system.

Aufgabe T20: Consider the inhomogeneous linear second-order ordinary differential equation

\[ -15u(x) + 3xu'(x) + x^2 u''(x) = 8x^{-3}, \quad x > 0. \]

(a) Find a real-valued fundamental system of the associated homogeneous differential equation.
(b) Find a particular solution by the method of variation of parameters. Determine the general solution of the inhomogeneous problem.

Tutorial date: Wednesday, May 21.