Worksheet No.7
Advanced Mathematics II

Exercise 31: For the inhomogeneous linear second-order ordinary differential equation
\[ x^2 y''(x) - \frac{3}{2} x y'(x) + y(x) = x^3, \]
determine
(a) the general solution of the homogeneous differential equation by means of reduction of the order. Use the fact that \( y_1(x) = x^2 \) solves the homogeneous problem.
(b) a particular solution and the general solution of the inhomogeneous differential equation by means of variation of the constants.
(c) a solution of the initial value problem with \( y(1) = \frac{17}{5} \) and \( y'(1) = \frac{21}{5} \).

Exercise 32: Determine the general solution in real form by the method of undetermined coefficients:
\[ y''(x) - 4y'(x) + 13y(x) = 2e^{2x} \sin(3x). \]

Exercise 33: Solve the initial value problem
\[ xu''(x) + 4u'(x) + 3u(x) = 3, \quad u(0) = 2, \]
with the power series method. For which \( x \in \mathbb{R} \) does the series converge absolutely? You don’t have to give the solution in an explicit form.

Exercise 34: Solve the initial value problem
\[ (2x - x^2)y''(x) + (1 - x)y'(x) = 0, \quad y(1) = 1, \quad y'(1) = 0 \]
with the power series method.

Exercise 35: Determine the general solution of the differential equation
\[ x^2 y''(x) + x^3 y'(x) - 6y(x) = 0. \]
Use the generalized power series representation \( y(x) = \sum_{k=0}^{\infty} a_k x^{k+\lambda} \).

Due date: Please hand in your homework on Wednesday, June 4, 09:30.
Tutorial 6  
Advanced Mathematics II

**Aufgabe T21:** Let the following differential equation be given
\[ x^2 y''(x) - 2xy'(x) + 2y(x) = x^3 \ln x, \quad x > 0. \]
The corresponding homogeneous differential equation has a solution of the form \( y(x) = Ax + B. \)

(a) Determine the general solution of the homogeneous problem by reduction of order.
(b) Determine a particular solution of the nonhomogeneous problem by variation of constants.
(c) Solve the initial value problem of the nonhomogeneous differential equation with \( y(1) = y'(1) = 1. \)

**Aufgabe T22:** Solve the initial value problem by the method of undetermined coefficients:
\[ u'''(t) + 4u'(t) = -2e^{-t}, \quad u(0) = 0, \quad u'(0) = 0, \quad u''(0) = 2. \]

**Aufgabe T23:** Determine the general solution of the homogeneous linear second-order ordinary differential equation
\[ u''(x) + x^2 u'(x) + 2xu(x) = 0 \]
using a power series ansatz near \( x_0 = 0 \) and specify the domain of convergence.
Remark: One of the two fundamental solutions can be expressed in terms of elementary functions.

**Aufgabe T24:** Solve the differential equation
\[ (2 + x)y''(x) + y'(x) = 1 \]
by means of an ansatz in power series form with center of expansion \( x_0 = 0 \) and determine the domain of convergence.

**Tutorial date:** Wednesday, May 28.