Exercise T4: Consider the vectors $u = (-2, 3, 1, 5)^\top, v = (1, 1, 2, 3)^\top, w = (-7, 3, -4, 1)^\top \in \mathbb{R}^4$.
(a) Construct the linear combinations $u + v + w$, $u - 3v + w$, $3u - 2v$, $2u - 3v - w$.
(b) Show that every pair of the vectors is linearly independent.
(c) Are all three vectors linearly independent? Determine the dimension of $\text{span}\{u, v, w\}$.

Exercise T5: Verify that the three vectors
$$u = (1, 2, 0)^\top, \quad v = (0, 1, 0)^\top, \quad w = (1, 1, 1)^\top$$
constitute a basis of $\mathbb{R}^3$. Express the vectors
$$a = (1, 2, 3)^\top \quad \text{and} \quad b = (3, 2, 2)^\top$$
as a linear combination of $u, v, w$. Determine the coordinates of $a$ and $b$ with respect to the basis $B$.

Exercise T6:

(a) Check in each case whether the given vectors are linearly independent.
(i) $u = (1, 1, 0)^\top$, $v = (1, 0, 1)^\top$, $w = (0, 1, 1)^\top$;
(ii) $u = (1, 2, 3)^\top$, $v = (2, 3, 4)^\top$, $w = (3, 4, 5)^\top$;
(iii) $u^1 = (5, 0, 5, -4)^\top$, $u^2 = (0, 5, -5, -3)^\top$, $u^3 = (5, -5, 10, -1)^\top$, $u^4 = (-4, -3, -1, 5)^\top$.
(b) For which $\alpha \in \mathbb{R}$ are the vectors $(2, 1, 3)^\top$, $(1, -1, 2)^\top$ and $(-\alpha, 4, -3)^\top$ linearly dependent? For these values of $\alpha$ express the third vector as a linear combination of the first and the second vector.