Problem Sheet 1

due date: 25.10.2011

Problem 1: Show that

1. $\sin(h) = O(h)$ as $h \to 0$
2. $e^{-1/h} = o(h^q)$ for any $q \in \mathbb{R}$ as $h \to 0+$
3. $-3n^2 + 200n - 20 = O(n^2)$ as $n \to \infty$
4. $\tan(x) = O(1/(x - \frac{\pi}{2}))$ as $x \to \pi/2$

Problem 2: Solve the Poisson equation $-\frac{\partial^2 u}{\partial x^2} = f(x)$ for $x \in (0,1)$ with the boundary conditions $u(0) = u(1) = 0$ and $f(x) = \sin(\pi x) + \sin(2\pi x)$ using the second order centered finite difference method. Preferably use Matlab and store the matrix K_h in the sparse format. Compute the approximate solution $\tilde{U} = (\tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_n)^T$ for $h = 1/10, 1/20, 1/40$, and $1/80$ and using the exact solution $u(x) = \frac{1}{\pi^2}\sin(\pi x) + \frac{1}{4\pi^2}\sin(2\pi x)$, plot the error $\|\tilde{U} - (u(x_1), \ldots, u(x_n))^T\|_{l^\infty}$ as a function of h. Does your error decrease like const.h^2?

Solve the problem also with $h = 1/10000$.

Matlab commands which can be useful: linspace, spdiags, \, loglog.

Problem 3:

Solve the stationary heat equation

$$\frac{\partial^2 u}{\partial x^2} = 0, x \in (0,1)$$
$$u(0) = 0, u(1) = 1$$

exactly and then numerically using the second order centered finite difference method with $h = 1/100$.