Institute for Applied and Numerical Mathematics 3 — Tomáš Dohnal
Karlsruhe Institute of Technology — dohnal@kit.edu
Introduction to the Numerics of PDEs — WS 2011/12

Problem Sheet 3

due date: 23.11.2011

Problem 1: Show that the matrix

$$A = \begin{bmatrix} 2 & -1 & 0 & & \cdots & 0 \\ -1 & 2 & -1 & & & 0 \\ 0 & -1 & 2 & -1 & & 0 \\ \vdots & & & \ddots & & \vdots \\ 0 & & & -1 & 2 & -1 \\ 0 & \cdots & & 0 & -1 & 2 \end{bmatrix} \in \mathbb{R}^{n\times n}$$

has the eigenvectors

$$x^k = \Big[\sin\big(\frac{jk\pi}{n+1}\big)\Big]_{j=1,\dots,n} \in \mathbb{R}^n,$$

with the corresponding eigenvalues $\lambda_k := 4\sin^2\big(\frac{k\pi}{2(n+1)}\big)$ for $k = 1, ..., n$.

Problem 2: Let K_h be the $n_1 n_2 \times n_1 n_2$–matrix corresponding to the five point stencil for the negative 2D Laplacian $-\Delta$ with zero Dirichlet boundary conditions on a rectangular domain, rowwise ordering, and an equidistant grid with $h_1 = h_2 =: h$. Using the Matlab command 'eig', check numerically that the $n_1 n_2$ eigenvalues are

$$\frac{4}{h^2}\left(\sin\left(\frac{\mu\pi}{2(n_1+1)}\right)^2 + \sin\left(\frac{\nu\pi}{2(n_2+1)}\right)^2\right), \quad 1 \le \mu \le n_1,\ 1 \le \nu \le n_2.$$